

Utilizing Geometric Mean in Proportional Fair Scheduling: Enhanced Throughput and Fairness in LTE DL

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Abstract—In this paper we address the challenge of multiuser scheduling in the downlink of Long Term Evolution (LTE) by bringing innovation in the concept of proportional fairness. We plug geometric average into the selection criterion of User Equipments (UEs) as opposed to widely used arithmetic average. We prove that the geometric average throughput converges to the solution of an ordinary differential equation, a similar result shown in the convergence of arithmetic average based Proportional Fair Scheduling (PFS). Extensive simulation results show that geometric average of UEs throughputs converge faster than arithmetic mean. This feature enables throughput increase in the cell while guaranteeing better fairness and Quality of Service (QoS) and imposing low Block Error Ratio (BLER). The superiority of the proposed algorithm is demonstrated by comparing the performance of the proposed scheduler with well-known standard schedulers under different simulation environments.

Index Terms—LTE, OFDMA, multiuser scheduling, geometric average throughput, fair scheduling, high throughput, stochastic approximation

I. INTRODUCTION

Orthogonal Frequency-Division Multiple Access (OFDMA) technology has been widely adopted as downlink radio access technology mainly due to its robustness to interference and multi-path fading and flexibility in resource allocation [1]. OFDMA-based networks such as LTE exploit frequency, temporal and multiuser diversity that this flexibility enables to achieve high system capacity. Resource grid (or time-frequency grid) of OFDM is divided into several Resource Blocks (RBs), which consists of 12 subcarriers for time duration of $0.5ms$. The allocation of RBs is governed by a centralized entity called eNB. The User Equipments (UEs) feed back on the Channel Quality Indicator (CQI), that corresponds to a specific code rate- modulation order, to base station (eNB) to maximize system throughput. The eNB can leverage this information to assign resources to the UEs. The objective of the scheduling entity embedded in the eNB is to schedule UEs in a way that the system will maximize the system throughput, while acting fair towards UEs and not penalizing UEs with low data demands.

In our work, we propose a novel algorithm for scheduling of downlink resources with the objective of overcoming the performance limits of the well-known existing solutions. Performance of Proportional Fairness Scheduling (PFS) [1] and

Best Channel Quality Indicator (BestCQI) [2] are severely impaired when UEs experience low Signal-To-Noise-Ratio (SNR) values, and this leads to increased Block Error Ratio (BLER) and therefore lower system throughput. Our objectives are therefore to enhance overall system capacity, Block Error Rate (BLER), fairness, and Quality of Service (QoS) of OFDMA-based wireless systems such as LTE. *The proposed algorithm re-designs the concept of proportional fairness, by using the geometric mean of transmitted data, instead of using the arithmetic mean, exploiting the fact that the former converges much faster than the latter.* In this way, the proposed solution initially behaves similarly to the BestCQI, and then later incorporates fairness into its decisions.

The contributions of this work can be summarized as follows:

- We have identified the performance limits of PFS for OFDMA-based downlink systems characterized by high frequency selective fading, in which channel conditions may not be evaluated properly by UEs.
- We have proposed an algorithm for scheduling of downlink resources applicable to centralized OFDMA-based networks, which overcomes the performance of well known PFS and BestCQI existing solutions in terms of system throughput, BLER, QoS, and fairness.
- We have proven that the geometric averages of UEs' throughput converge to the solution of an ordinary differential equation (ODE). ODE has a unique equilibrium. The existence of a unique equilibrium of ODE determines the throughput of each user and hence the delay [3].
- We have extensively evaluated the performance of the proposed algorithm and demonstrated its general validity by testing it under multiple network conditions.

Proposed algorithm was introduced in [4]. In this paper, in addition to analyzing the performance of the algorithms in different settings and environments, we have also shown convergence of the proposed algorithm. Moreover, another performance metric, average waiting time, has been illustrated.

The rest of the paper is organized as follows. Section II presents the related works. Section III describes the system model and the formulation of the problem. Section IV briefly presents the weak convergence of the geometric average.

Section V discusses the proposed algorithm for the downlink scheduling. Simulation parameters, results and discussion presented in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK

PFS algorithm was devised for single Channel CDMA/HDR system. When there is a single carrier whose channel state information (CSI) is computed reliably, PFS is expected to perform well; however, in typical LTE system where subcarriers' CQIs are not reported precisely, PFS performance degrades. Kushner et al. [3] have shown that UEs' mean throughput converges to a solution of an ordinary differential equation (ODE) in PFS. Because of slow convergence, PFS tends to assign chunk of RBs to a single UE with low achievable data transmission demand. The UE fails to demodulate the transmitted data successfully due to poorly computed CQI. In subsequent TTIs, PFS assigns RBs to the same UE until the UE's ratio of current instantaneous rates to mean throughput does not maximize the utility function for given RBs. Thereby, PFS accomplishes fairness among UEs in terms of individual throughput but results in poor radio resources management. BestCQI favors the UEs with the largest CQI values; thus, it achieves high system throughput by trading off fairness towards UEs characterized by non-favorable channel conditions.

Schedulers can be divided in two groups: (i) Channel-aware/QoS-unaware (ii) Channel-aware/QoS-aware. In [5]–[8] authors proposed schedulers that are Channel-aware/QoS-unaware. In [5] an UE with maximum CQI values is assigned to RBs and the selected UE is not permitted to be scheduled until the end of that TTI; if all UEs are scheduled and there are still free RBs, the scheduler takes all UEs into consideration. Authors have executed their proposed algorithms in a setting where all UEs share same SNR. Generalized PF (GPF) method is developed in [6] by introducing two parameters ξ and ψ .

$$\arg \max_{\substack{i \leq N \\ j \leq m}} \frac{[r_i^j(t+1)]^\xi}{[\theta_i(t)]^\psi} \quad (1)$$

GPF encompasses both Best-CQI and PFS simply by tweaking ξ and ψ values in (1). The new parameters ξ and ψ must be able to dynamically adjust themselves depending on current cell load to be useful. Similar adaptive approaches capable of tuning fairness levels dynamically proposed in [7] and [8]; however, computing such parameters takes longer than scheduling.

In [9]–[12] authors proposed schedulers that are both Channel-aware and QoS-aware. QoS is managed by introducing a set of QoS parameters to guarantee either data rate or packet delivering delay. A Guaranteed Data Rate (GDR) approach is proposed in [10]. A decoupled time and frequency scheduling as proposed in [6] strategy is adopted. The Time Domain Priority Set Scheduler (TD-PSS) divides UEs into two sets. UEs below Target Bit Rate (TBR) comprise Set 1 and are given high priority. UEs in the first set scheduled based on the Blind Equal Throughput (BET) metric whereas other UEs in the second set are selected according to PFS metric.

Such approach that only concerns with GBR will have delay problem and therefore high Packet Loss Rate (PLR).

A Scheduler that bounds packet-delivering delay is proposed in [12]. Authors proposed Delay Prioritized Scheduler (DPS) that selects UEs based on packets' head-of-line (HOL) proximity to delay threshold. UEs with lowest values, which means whose packets are about to drop, are scheduled first. This way low PLR of less than 1% is achieved; yet, this may result in low cell throughput.

III. SYSTEM MODEL AND PROBLEM FORMULATION

The system comprises a cellular network with N UEs associated to a single eNB operating on a single channel whose bandwidth is divided into m orthogonal narrowband subcarriers. Each UE provides feedback on the averaged CQI of all subcarriers to the eNB [13]. At Transmission Time Interval(TTI) t , the eNB knows the CQIs for subsequent TTI for each UE. If UE i is selected in TTI t on RB j then it transmits $r_{i,t}^j$ unit of data where $\{r_{i,t}^j, t \leq \infty\}$ is bounded. Let $I_{i,t}^j$ be the characteristic function. $I_{i,t}^j = 1$ if RB j is assigned to UE i at time slot t and is equal to 0 otherwise. $\tilde{Q}_{i,t}$, which is defined as follows, represents total data unit that UE i transmitted over all RBs at TTI t :

$$\tilde{Q}_{i,t} = \begin{cases} Q_{i,t}, & \text{if } Q_{i,t} > 0 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

and

$$Q_{i,t} = \sum_{j=1}^m r_{i,t}^j I_{i,t}^j. \quad (3)$$

One definition of the throughput for the UE i up to time t is the geometric mean:

$$\beta_{i,t} = \sqrt[t]{\prod_{l=1}^t \tilde{Q}_{i,l}} \quad (4)$$

At each TTI, each RBs is assigned to only one UE according to the following formula:

$$\arg \max_{\substack{i \leq N \\ j \leq m}} \left\{ \frac{r_{i,t+1}^j}{d_i + \beta_{i,t}} \right\} \quad (5)$$

d_i is a negligible positive constant to prevent division by zero when the current throughput is zero. Transmitted data rates can be different than r_i or data may not be successfully transmitted due to poor channel condition. We will denote data unit conveyed with success with r_i^* from now on. For a given window size α and discount factor $(1 - \frac{1}{\alpha})$, the discounted throughput is defined recursively as:

$$\beta_{i,t+1} = \beta_{i,t}^{(1-\frac{1}{\alpha})} r_i^{*\frac{1}{\alpha}} \quad (6)$$

In the case of ties a random UE is selected among UEs with the highest ratio. In order to maximize system throughput, (5) must be maximized for every single RB j at each TTI [3].

IV. WEAK CONVERGENCE OF THE GEOMETRIC AVERAGE

In this section, we prove that the geometric average $\beta_{i,t}$ weakly converges to the limit points of the *mean* ODE. For simplicity, we only consider the case where there is only one RB, $m = 1$, and the superscript j is dropped off in $r_{i,t}^j$ and $I_{i,t}^j$ for notational abbreviation in this section. The weak convergence for general m RBs is similar. Taking the logarithm of (2), we get

$$\log \beta_{i,t} = \sum_{l=1}^t \frac{\log \tilde{Q}_{i,l}}{t} = \sum_{l=1}^t \frac{\log(r_{i,l})I_{i,l}}{t} \quad (7)$$

Then we obtain the two-term recursive relation similar with equation (1.2) in [3],

$$\log \beta_{i,t+1} = \log \beta_{i,t} + \epsilon_t \left[\log(r_{i,t+1})I_{i,t+1} - \log \beta_{i,t} \right] \quad (8)$$

where $\epsilon_t = 1/(t+1)$.

Similarly as in [3], we define (prime stands for matrix transpose)

$$\hat{\beta}_t = \left[\log \beta_{1,t}, \dots, \log \beta_{N,t} \right]' \quad (9)$$

and

$$R_t = \left[\log r_{1,t}, \dots, \log r_{N,t} \right]' \quad (10)$$

Since the usual stochastic approximation asymptotic analysis of (8) employs continuous time interpolations, for each t , let us define shifted process $\hat{\beta}^t(\cdot) = [\hat{\beta}_1^t(\cdot), \dots, \hat{\beta}_N^t(\cdot)]'$ such that $\hat{\beta}^t(0) = \hat{\beta}_t$ and for $l > 0$

$$\hat{\beta}^t(s) = \hat{\beta}_{t+l}, \text{ for } s \in \left[\sum_{k=t}^{t+l-1} \epsilon_k, \sum_{l=t}^{t+l} \epsilon_l \right] \quad (11)$$

where the empty sum is zero. Since $\hat{\beta}^t(\cdot)$ begins at t , the behavior of $\hat{\beta}^t(\cdot)$ as $t \rightarrow \infty$ is that of $\hat{\beta}_t$ as $t \rightarrow \infty$.

To prove the weak convergence, we need some assumptions similar as in [3].

A1: Let the ξ_t denote the past $\{R_l : l \leq t\}$. For each i, t, ξ_t ,

$$h_{i,t}(\beta, \xi_t) = E_t \log(r_{i,t+1})I_{\left\{ \frac{r_{i,t+1}}{d_i+\beta_i} \geq \frac{r_j,t+1}{d_j+\beta_j}, j \neq i \right\}} \quad (12)$$

is continuous in $\beta \in \mathbb{R}_+^N$ where $\beta = [\beta_1, \dots, \beta_N]'$ is used as the canonical value of $[\beta_{1,t}, \dots, \beta_{N,t}]$ and I represents the characteristic function. Let $\delta > 0$ be arbitrary, then in the set $\{\beta : \beta_i > \delta, i \leq N\}$, the continuity is uniform in t and ξ_t . Assume R_t is bounded.

A2: $\{R_t, t < \infty\}$ is stationary. Define $\tilde{h}_i(\cdot)$ by the stationary expectation

$$\tilde{h}_i(\beta) = E \log(r_i)I_{\left\{ \frac{r_i}{d_i+\beta_i} \geq \frac{r_j}{d_j+\beta_j}, j \neq i \right\}} \quad (13)$$

where r_i is used as the canonical value of $r_{i,n}$. Then by following the similar proof in [3] we have the following theorem hold:

Theorem 1: Given the two-term recursive relation (8), conditions (A1) and (A2), for any initial condition, $\hat{\beta}^t(\cdot)$ converges weakly to the set of limit points of the solution of the ODE:

$$\dot{\hat{\beta}}_i = \tilde{h}_i(\hat{\beta}) - \hat{\beta}_i, \quad i = 1, 2, \dots, N. \quad (14)$$

And the limit point $(\bar{\beta})$ is unique, irrespective of the initial condition. So the process $\hat{\beta}^t(\cdot)$ converge to $\bar{\beta}$ as $t \rightarrow \infty$.

proof of the Theorem 1: Since following the same idea of [3], we omit the main proof but only check that $f(x) = \tilde{h}(x) - x$ satisfies the K -condition [14]. If $x \leq y$, $x_i = y_i$, then

$$\begin{aligned} f_i(x) - f_i(y) &= \tilde{h}_i(x) - \tilde{h}_i(y) \\ &= E \log(r_i) I_{\left\{ \frac{r_i}{d_i+y_i} \geq \frac{r_j}{d_j+x_j}, j \neq i \right\}} \\ &\quad - E \log(r_i) I_{\left\{ \frac{r_i}{d_i+y_i} \geq \frac{r_j}{d_j+y_j}, j \neq i \right\}} \end{aligned} \quad (15)$$

notice that $\frac{r_i}{d_i+y_i} = \frac{r_i}{d_i+y_i}$ but $\frac{r_i}{d_i+y_i} \geq \frac{r_i}{d_i+y_i}$, then

$$I_{\left\{ \frac{r_i}{d_i+x_i} \geq \frac{r_j}{d_j+x_j}, j \neq i \right\}} \leq I_{\left\{ \frac{r_i}{d_i+y_i} \geq \frac{r_j}{d_j+y_j}, j \neq i \right\}} \quad (16)$$

which leads to $f_i(x) \leq f_i(y)$, the K-condition.

V. FAHT ALGORITHM

Two different version of the Fair Allocation High Throughput (FAHT) Algorithm- FAHT₆₀ and FAHT₁₀₀ - are illustrated here. The steps for FAHT₆₀ and FAHT₁₀₀ are same. The

Algorithm 1 FAHT Algorithm

- 1: Get CQI values from each UE
 - 2: Compute the ratio of instantaneous data rate to average transmitted data rate for each RB for each UE
 - 3: **while** Number of RBs has not been assigned > 0 **do**
 - 4: Allocate RBs to the UE with maximum ratio
 - 5: Delete the RB that has been allocated
 - 6: **end while**
 - 7: Update average throughput values β for each UE
-

only difference is in calculating geometric mean. For FAHT₆₀ we use true transmitted data rate to calculate β ; however, to compute β in FAHT₁₀₀ we divide true transmitted data by 100 and add 1 to speed up the convergence.

A. Intuition behind FAHT

In the following, we discuss the intuition behind FAHT and why it performs better than PFS. Proof of the convergence rate is currently an ongoing work. We discuss simulation parameters of the environment Fig.1 represents in *simulation result* section. As seen, β_i values, for FAHT₆₀ and FAHT₁₀₀ (second and third images), converge fast and are very close to each other's. The ranges of Y-axis for both algorithms are much smaller then of PFS. That said, any small changes in the numerator (instantaneous data rate) of the selection criterion (5) results in selection of a different UE. However, for PFS, it takes 300 subframes for some UEs' mean throughputs to congregate around a value. Yet, those values are not that close till the end of the simulation. Therefore, even when big

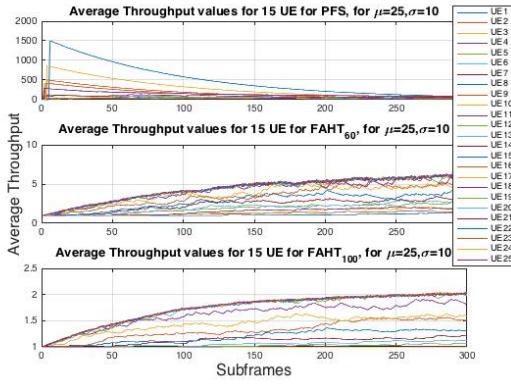


Fig. 1. Changes in the UEs average throughput values over time under *PFS*, *FAHT*₆₀, and *FAHT*₁₀₀ from top to bottom, respectively.

changes happen in numerator of equation (1.6) in [3], it does not affect who will be selected next.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Number of User Equipments (UEs)	25
Channel Bandwidth	1.4 MHZ
Simulation length	300 subframes
Transmit Mode	MU-SISO
Scheduling Algorithms	BestCQI, PFS, <i>FAHT</i> ₆₀ , <i>FAHT</i> ₁₀₀

VI. SIMULATION PARAMETERS, RESULTS AND DISCUSSION

A. Simulation Parameters

The Vienna University's MATLAB based LTE link-level simulator [15] has been used to evaluate the performance of the proposed algorithm in comparison with PFS and BestCQI solutions. The simulations are performed for frequency-selective channels modeled by ITU, for Pedestrian-B (PedB) channels. Table I summarizes the main simulation parameters adopted.

B. Simulation Results

The scheduling algorithms are evaluated with regards to overall cell throughput, BLER, QoS and fairness. In fact, beside system throughput, BLER is also very important metric. Then, individual UEs' throughput, and allocated RBs quantify the fairness of each of the scheduling algorithms. Three different network environments are considered to demonstrate the superiority of the proposed algorithm over PFS and BestCQI performances. The first simulation scenario considers 25 UEs having SNRs ranging from 2 to 50 dB in 2dB steps. In the second scenario UEs' SNRs normally distributed with parameter $\mu=25$ dB/ $\sigma=10$. UEs undergo similar SNR value of 18 dB in the third scenario.

Fig.2 (a) and (b) displays the system throughput and BLER for the three scenarios respectively. The proposed algorithm performs better than PFS in three scenarios in terms of system

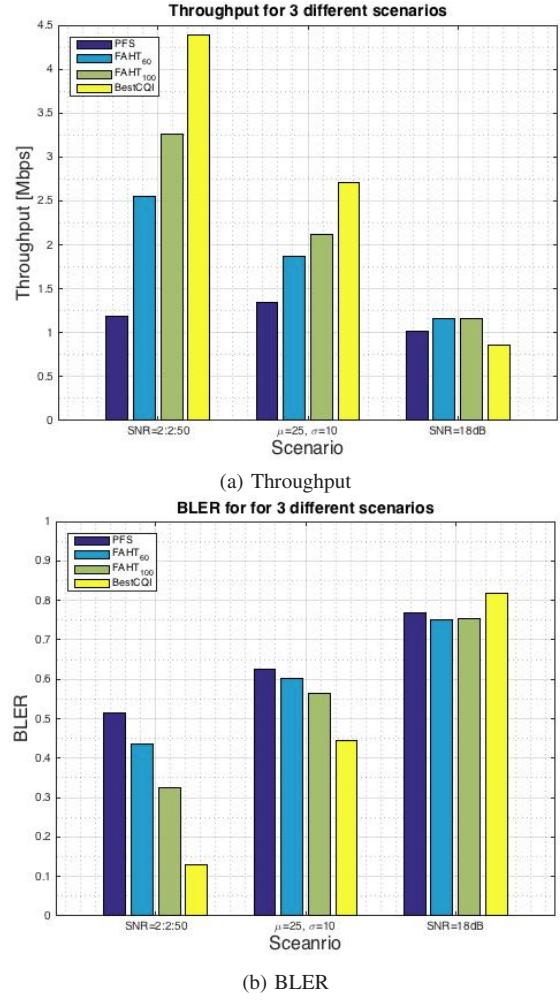


Fig. 2. : a) Cell throughput b) BLER of 25 UEs under 3 different scenarios

throughput. In the low SNR region, which is the third scenario, our algorithm outperforms BestCQI. Although BestCQI excels when the network is characterized by large variation of SNR experienced by UEs -first and second scenarios- this comes at the cost of compromising fairness. BLER performances of the schedulers are in accordance with their throughput performances. Fig.3 shows the throughput achieved by different UEs with each scheduler in second scenario. UEs' SNR values are sorted during the simulation. BestCQI achieves high throughput for UEs with favorable SNR but UEs with low SNR starve. Proposed algorithms favor UEs with high SNR but do not let starvation. Under PFS, differences between UEs throughput values do not change drastically. Considering the nature of wireless network applications, differences between UEs' throughputs are necessary to increase system throughput. For example an UE streaming high definition video should have higher throughputs than UEs receiving a voice over IP stream in order to be "proportionally" fair. Fig. 4 shows the number of RBs allocated to different UEs with different schedulers in the second scenario. Again, UEs with favorable SNR receive most of RBs in BestCQI. PFS assigns most of

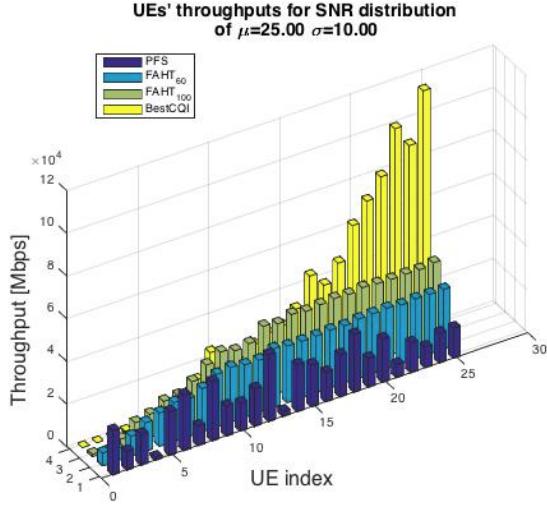


Fig. 3. Individual UEs' throughputs in second scenario $\mu=25\text{dB}$, $\sigma=10$.

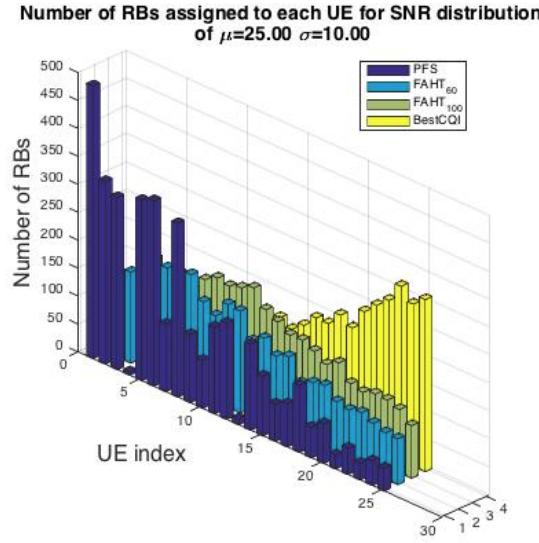
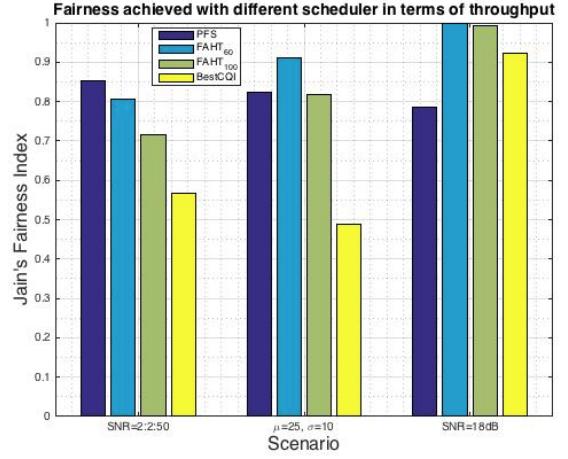


Fig. 4. Number of assigned RBs to each UE in second scenario $\mu=25\text{dB}$, $\sigma=10$.

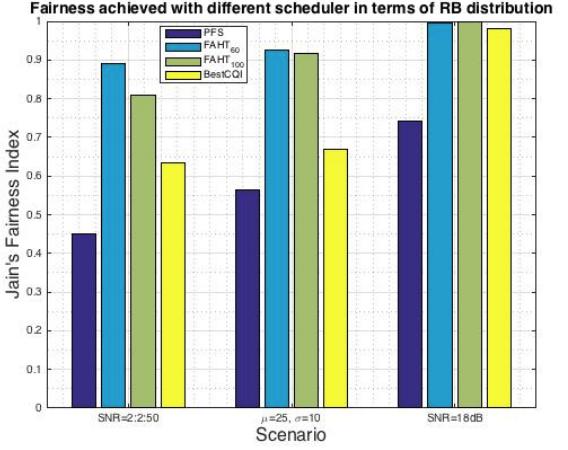
RBs to UEs with low SNR. Although proposed algorithm has a tendency to favor underdog, it is not as radical as PFS. There are two weaknesses of PFS we observe here: First, PFS sacrifices radio resources in spite of fairness provided to the single UE throughput. For example, UE_1 , who has the lowest SNR, receives almost 500 RBs to achieve what he achieves in Fig.3. Second, UE_4 at both Fig.4 and Fig.3 has the lowest statistics. This is because UE_4 is the UE whose average throughput converges in 300 subframes in Fig.1 at the top figure (UEs with blue legend). Therefore, it is never assigned any RBs except the first time it was scheduled.

We quantify fairness using Jain's fairness index [16]. BestCQI is not compatible with fairness neither in terms of throughput nor in RB distribution when UEs experience different SNR, respectively first and second scenarios. Proposed algorithms achieve highest fairness in all cases except in terms

throughput in first scenario yet as good as PFS. In accordance to Figure 4 PFS is not compatible with fairness at all in terms of RB distribution. The only time PFS achieves highest fairness is the first scenario in terms of throughput where UEs having SNRs ranging from 2 to 50 dB in 2dB steps.



(a) Fairness Index in terms of throughput



(b) Fairness index in terms of RBs distribution

Fig. 5. Fairness achieved with regard to a) Throughput b) Radio Resource Distribution with each scheduler in different scenarios

Another important limitations of both PFS and BestCQI we have discovered from the results of extensive experiments is this: The proposed algorithm tends to assign at most one RB to a single UE in low SNR conditions, whereas BestCQI and PFS tend to assign chunk of subcarriers to a single UE at each TTI. Since all RBs allocated to a given UE in any scheduling period will use the same modulation and coding scheme (MCS), frequency selective fading may cause high BLER and consequent performance loss in BestCQI and PFS for low SNR due to erroneous CQI (and consequently MCS) selection. It is obvious from the Fig5 (b) Best CQI is as fair as proposed algorithm in the third scenario where UEs experience same SNR of 18dB; however, proposed algorithm outperforms BestCQI with regarding to cell throughput.

Finally, Fig.6 illustrates average delay in packet delivering. We divide whole simulation, which are 300 subframes into

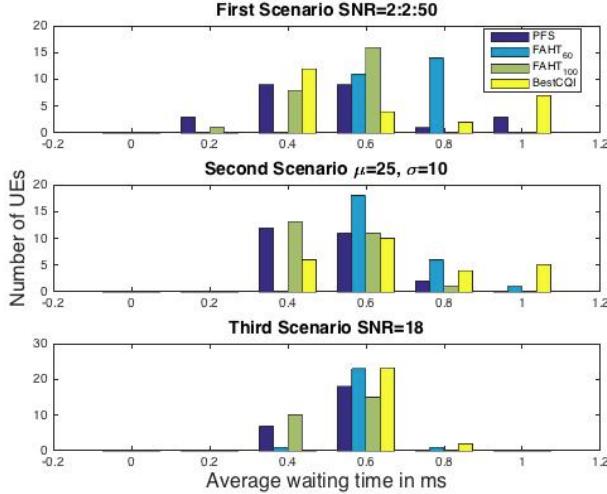


Fig. 6. Average waiting time to be scheduled with different schedulers in 3 different scenarios.

small intervals. The length of each interval is different than each other for each UE and within each UE; for example, for each UE, first interval starts from the first TTI and ends at the very first TTI that follows one or series of TTIs which that particular UE is not scheduled at; second interval starts from where first interval ends and so on. Average waiting time, for each interval, is determined by dividing number of TTIs the UE is not scheduled at to the length of that interval. Lastly, the mean of average waiting times is calculated by dividing the sum of these values by total number of intervals for each UE. Under proposed algorithm and PFS most UEs wait, on average, 4-6ms in every 10ms to be scheduled. This is way lower than threshold for HOL packet delay set in [12] which is 20ms. As for BestCQI some users are never scheduled in first and second scenarios.

VII. CONCLUSION

We proposed a novel downlink-scheduling algorithm for OFDMA downlink transmission and compare its performance with two well known algorithms, BestCQI and PFS. The performances of new schedulers are investigated with regards to system throughput, BLER, fairness. We have proven that the geometric average of UEs' throughputs also converge to the solution of an ODE and our experimental results show that proposed algorithm improves system throughput, BLER, and balances the use of shared radio resources among UEs. Extensive evaluations illustrated that the geometric mean throughput converge much faster than arithmetic mean.

Future research will address the proof of convergence rate and extension of the proposed algorithm to include advanced LTE features such as MU-MIMO, carrier aggregation (CA) as well as new application paradigms with different UE traffic characteristics such as Internet of Things (IoT) applications.

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