COMPLEXITY ANALYTICS AND POLICY
CAUTION AND OPPORTUNITY GOING FORWARD

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COMPLEXITY ANALYTICS AND PUBLIC POLICY: CAUTIONS AND OPPORTUNITIES GOING FORWARD

Introduction

The social sciences have been experiencing a new awakening under the bright light of complexity theory that has been shining on them and galloping through their intellectual space in recent years. The key to unleashing the power of complexity analytics for social good is to realize, accept, and adhere to the fact that the analysis of complexity is a state space proposition. As we proceed on this mission, there are two fronts on which we will need to exercise special care: 1.) the dimensionality of any imaginable social space is unimaginably infinite; and 2.) our knowledge and understanding of the topology of social space is at best embryonic

State space is a representation of all possible combinations of all possible values of all state variables, and the relationships among them. A point in state space is a complete description of the system at that point. Phase space is a snapshot in time of the simultaneous movement of the system through the values of all state variables. A mapping of phase space displays a manifold of phase lines which show the combined values of state variables where phase changes in the system occur. Phase space is also useful in that excluded regions of state space are displayed. Fractal geometry is used to describe the phase space of complex adaptive systems because it is the only mathematical language we know that can encompass the phase lines of such systems which do not overlap, do not repeat, and do not settle into a steady state.

A phase change is a discontinuous change in the state of the system. It indicates the crossing of a physical boundary between two different regions of phase space (one of those manifold lines referred to earlier). Phase changes are reversible, and moving from one phase to another fundamentally alters the energy relationships among the system components or agents. What phases a system can occupy is a function of the physicality of system. We are accustomed to solid, liquid, and vapor phases of elements; but there are also plasma, glass (colloidal), and crystalline phases. There seems to be a growing acceptance of phase change in social systems as associated with system collapse. How have we defined phase in these instances and what additional manifestations does that definition allow? Is it possible to imagine a definition of phase on a social space where a discontinuous change would lead to an energetically different, yet still ordered state. Are we looking at the right variables in our efforts to understand phase in social systems.

Phase space refers to currently accessible regions of state space. It captures the coupling of state variables Interaction among or coupling of system parameters could create excluded regions of state space. A phase space mapping would identify these excluded regions and their changes over time. Phase space maps the evolution of system properties with collective changes in the values of state parameters. In simple mechanical or chemical systems, points in phase space correspond to manifestations of physical characteristics like the arc of a pendulum, or the liquid, solid, or gaseous state of water. In social systems this correspondence is not so clear both because of lack of precision in the definition of system parameters, and because of the historical practice of representing social systems as linear and static. A phase change is not just a change of position in state space. A phase change is the crossing of a boundary in phase space that marks the limits of the combination of values of state variable associated with the manifestation of specific system behavior and properties. Inside those boundaries. The system can experience a change in state without undergoing a phase change. The familiar example of phase change is the transition of water from vapor to liquid to solid.   In regional development would the transition from agricultural to industrial to informational to networked stand the definition of phase change given here? These are often referred to a phases of economic development, but in what sense would such developments be reversible?

The Centrality of Phase Space

Understanding the dynamics of phase in social systems is the sine qua non of our ability to develop and implement meaningful and effective policies in the realm of complex adaptive social systems. Researchers have, correctly in my opinion, focused on events involving "spontaneous disassembly", like stampedes and riots, because of the theoretical observation that a phase change involves a radical and spontaneous change in system structure. It is important to remember, however, that change in system structure itself has an underlying cause: the crossing of a physical boundary, a phase line, in phase space. Phase lines appear where bifurcations occur. A bifurcation in the analysis of complexity is not the simple splitting of a whole into two parts, or the branching of a network. A bifurcation is the doubling of the number of values that a state variable can occupy. Bifurcations occur under the influence of a “bifurcation parameter” the value of which is system specific and reflective of the physicality of the system. A change in its value is usually associated with a change in the total energy available to the system. As the value of that parameter changes continuously, the system can at certain points undergo discontinuous changes, radically altering the behavioral properties of the system. The same system can go from simple and stable to complex and chaotic, depending upon the value of this driving parameter. As the value of this parameter changes the system becomes unstable until it bifurcates and becomes once again stable and the process begins again. This process can produce an infinite series of bifurcations; however, the range of the value of the bifurcation variable over which the system remains stable diminishes as the value of this parameter increases, so there is a limit to the number of times a bifurcation can occur that will result in stable states. This limit is a function of the physicality of the system, but systems where it goes beyond three are rare. Beyond that point, the system is chaotic.

Identifying the bifurcation parameter would seem to be the prime objective of research in complex social systems. It would seem to me that enormous progress could be made on this question by a retrofit of existing data on social systems. Repositories are full of data on social systems. Much of it has been collected under the assumption that the relationships among the variables are time invariant and linear, which we now know is not true; but so much is known about these systems already, that taking another look simply to determine the existence of bifurcations could be very productive. The modern frenzy over Big Data is also fortuitous. Care would have to be taken in mining these data, though, because this would not be an exercise in statistics. Some Big Data databases are simply indiscriminant amalgams of huge amounts of data from wherever. In looking for bifurcations, the data have to be FROM the systems being analyzed so matching data to source is another bit of retrofitting that would be required.

What I am suggesting is that searching existing databases and social systems research archives for bifurcation occurrences could lead to new and exciting discoveries about the phase space of social systems. It could also uncover volumes about orthogonality and other aspects of the topology of social space, and help unravel the mysteries of attractors giving us access to points of stable and unstable equilibrium in the phase space of complex adaptive social systems with all the associated predictive value.

Modeling Complex Adaptive Social Systems

Genetic modeling is a state space-based tool of complexity analytics that is very satisfying to the purist as it requires doing very little damage to the assumptions of complexity science. Modeling complex systems is a conundrum, however, especially in the case of complex social systems analysis, in that the researcher is in essence creating the dynamics being observed and attributing to the results some meaning for real world complex systems. This is particularly problematic for fitness landscapes because such landscapes cannot help but be arbitrarily carved out of segments of state space with little or no discipline imposed by an understanding of the underlying topology. In fact, the topology is created by the researcher's a priori understanding of the nature of the system. Complexity science treats the system and its environment as one interactive phenomenon from a dynamic state space perspective. In order to correctly integrate system components, we need to understand the topologies of the spaces the various system components occupy.

The topologies of physical spaces are well understood; however our awareness and understanding of the topology of social space is in its embryonic stage at best. The author of one book on the topology of social space has actually asserted that social space is flat (Krieger, 2014) . Topology is a mathematical concept. No data or analyses were presented to support this conclusion, and given that linear regression, which assumes a linear relationship among the dimensions of the space, is the dominant tool in social systems analysis, the conclusion is, at best, circular. Other efforts to understand the topology of social spaces have focused on networks and have associated the topology of social spaces with network topology. That direction is interesting and promising.

This criticism is less problematic for agent based models. While it is true that agents, environments, and rules of interaction are selected, designed, and defined by researchers, the map between these elements and the real world is much more apparent. Conceptually, however, the grids on which agents move in agent-based models are usually mathematically defined distributions that do not begin to capture the deep structure of real social space. Underlying social networks on the other hand *are* maps of reality and they provide powerful insights into the diverse topologies of social space. These topologies are fundamentally relational and accommodative of the embedded deep structure of social space. It is not clear what techniques would support the translation of such relational dynamics to mathematically defined environments, how these topologies affect fitness landscape potentials, and what that means for the movement of agent across them. An extremely useful contribution to the field would be a research program aimed at understanding whether or not and if so how these three techniques might be combined.

In evolutionary models the important system characteristics are not averages. They are adjacencies. The coding of state space is binary, and under normal (non-chaotic) conditions, evolutionary movement is between adjacent states. In evolutionary modeling of social systems, the art will be in the specification of state space and in learning to appreciate the definition, characteristics, and dynamics of social systems as emergent. Regression analysis of social phenomena is problematic because of our lack of understanding of the topology of social space. In state space analyses the topology is moot because the system evolves that topology and evolves in that topology all the time, in real-time and over time. The topology of the space is created by the evolutionary rules.

Evolutionary models have proven extraordinarily useful. Stuart Kaufman's fitness landscapes are an important example of this tool. According to Kaufman, the origin of order is evolutionary movement in state space. If at the level of the organism its state is represented by the bit string

0011010100100011

That evolutionary change is incremental and random would be captured by a single change (called a mutation) in any single bit to something like

0011010101100011

where the two are called "nearest neighbors" in state space, and the "neighborhood" consists of all states that can be represented by a one bit change from the original. If you add to this the understanding that the bits represent genes or characteristics of the organism and the bit string represents a chromosome, it becomes apparent that a change in a bit carries with it a change or an evolution in the physical characteristics of the organism.

Our bodies are governed by genes which are embedded in chromosomes which are embedded in cells which are embedded in tissue which are embedded in organs . . . Thinking about the many different types of tissue that make up our bodies -- liver tissue, brain tissue, heart tissue, nerve, skin, muscle, . . . -- and all of the interactions, processes, chemicals, and bodily fluids that combine to support life, makes it hard to believe that it is all orchestrated by random fluctuations at the level of the gene. Turns out that this unfathomable process repeats itself in the emergence, development, and functioning of all complex adaptive systems.

Starting with the bit string process described above, Kaufman has fashioned a model which captures that process called the NK Fitness Landscape Model. The evolutionary process is obviously far more complex (and complicated!) than I have describe, but there is just one more piece of the story that needs to be added to enable us to move forward. The state space of the bit string consists of every possible combination of zeros and ones of length N (the length of the string -- this is the N in Kaufman's NK Fitness Landscape model). Since there are two possible values at each position, there are 2^N possible configurations of this state space. This number grows very large very quickly, and it is only the increasing availability of increasingly fast computers with increasingly large and inexpensive storage space that makes these kinds of models possible. The collection of all such configurations is in this case, 65,536 (2^{16}) different strings representing all possible unique configurations of zero and one in a16 bit long string.

The process of evolution is random. It is, however, possible for it to be constrained. Constraints are created by so-called "alleles" which are combinations of genes that tend to evolve together, like brown skin and curly hair or blond hair and blue eyes. The extent to which the evolutionary process is constrained is captured by the K in Kaufman's NK Fitness Landscape model. Fitness is a measure of the probability of evolutionary success. Fewer constraints lead to fewer bumps in the road and a smoother landscape. A highly constrained process creates a "rugged" landscape. Peaks and valleys in the landscape represent high and low fitness values. So a fitness landscape is a surface over state space that offers a measure of the probability of evolutionary success at each point.

Networks

Natural laws are neither necessarily correct nor immutable as given. They are nothing more than the best approximation we are able to make at the moment. The earth is careening around the sun, the sun is careening around the galaxy, and the galaxy around the universe all in coupled orbits held in balance by "gravity". There is no known universal fixed point of reference in time or space[1]. Everything is defined relative to something else. Our scientific systems of thought, whether explicitly or not, are fundamentally interdependent and relational.   Mass has no independent existence and is technically defined as energy at rest. Measurement of time and space depend on your frame of reference. The effects of major force fields can be simply transformed away, and who knows what the recent validation of dark energy and dark matter portend?

Complexity science is completely comfortable with this set of circumstances, and most sciences have begun converting themselves to a complexity motif (Quantum Hamiltonian Complexity, Complex Systems Biology, Mathematical and Computational Chemistry, . . .). We have just taken a look at fitness landscapes, one of the most important tools of complexity analysis, and we have seen that it is profoundly relational, providing a mechanism for conducting searches among "nearest neighbors". Fitness landscapes are becoming a generalized optimization tool used across industries from finance to materials science to quality control. Adjacency is the star of the fitness landscape story, and it is a curious circumstance that a mathematical description of the biological process of evolution should find applicability in such diverse arenas.

Networks are another one of nature's favorite ways of operating: ant colonies, circulatory systems, nervous systems, highways, railroads, the Internet . . . Yes, these latter are man-made; but remember we are a part of nature (ant colonies at a larger scale!). Network theory is also an important tool of complexity analysis, and it is, by definition, relational. It is interesting that the adjacency matrix (or adjacency list) is the starting point for the rendering of a network. Networks are maps or graphs of connected relationships. They are garnering much attention lately, and theory has built up around them describing what their structure says about the nature of the underlying relationships being mapped.

Networks consist of connected edges and nodes. Network structures include random, scale-free, small-world, hierarchical, . . . Nodes and links have degree, density, betweeness, centrality, connectivity. . . Links can combine through nodes to create paths and distances, network circumferences . . . All of this is the "stuff" of network theory and combines to create powerful tools to control the spread of disease, trace terrorist activity, identify and skirt potential network failure, design proteins, and in some cases, just have fun! Its importance here, though, is that every node in every network is an adjacency, and by Kaufman's model, a place where something evolutionary and developmental can happen. Research can be published; electricity can flow; children can be conceived; treaties can be made; proposals can be shared; ideas can go forward; . . . Conversely, without adjacency, nothing can happen. There are even those who have argued that the topology of the network IS the topology of the space it spans. For example, your motivation and behavior is very different in a conflict space that involves your

[1] Though the dark matter related discovery that the universe is expanding faster suggests that we might be able to project that process backwards to the physical location of the Big Bang.

child than in one that does not. The very shape of that space and its force upon you is determined by your relationship to those involved and that force is something that you physically feel through to your bones.

The Deep Structure of Social Space

Generations of emergence has created an embedded history in the psyches of individuals, the structure and operations of institutions, and the tenets of cultures that plays a role in constructing their ability and willingness to participate new possibilities, even in shaping their perceptions of what new possibilities are. Kaufman has shown that change in complex systems is evolutionary and incremental. That is, given a system in a particular state, moving that system to a different state is most likely to be accomplished state by adjacent state. Understanding the structure of social space is crucial to understanding the dimensionality of adjacency, which dimensions are most important in a given change process, and which participants are where along each dimension.

Where this discussion is going is toward the suggestion that "adjacency analysis" be constructed for all public policy enterprises. Just as adjacency portends positive evolutionary possibilities, so also does widespread lack of adjacency in intervention design provide an explanation for failure. Haiti, race relations in the United States, failed development strategies, US training of troops in Iraq -- these all suffer from the same dislocation: there is insufficient adjacency among the states of all the players given the scope of the enterprise. Adjacency analyses will minimize the distortions caused by our lack of knowledge about the interactions of the diverse topologies of social space because movement will be from state to state in the case of fitness landscapes and along topological lines in the case of network analysis.

An example might help to clarify the power of this approach. A group of Haitian immigrant boys in Brooklyn displaced by the 2010 earthquake in Haiti were having severe behavioral and academic problems in school. They were brought into a community of other Haitian teenage boys with whom the affinity was that they were all Haitian, not that they had all gone through the earthquake. A normalcy evolved, they were able to rebalance themselves emotionally and their behavioral and academic problems disappeared.

Another story. There is a Harvard-MIT Atlas of Economic Complexity which, operating from what they call the "product space" ranks the countries of the world according to economic complexity. The index they use measures the complexity of the products the countries produce. Their objective is to enable countries to be more efficient in the economic development choices they make. If your country is currently functioning at complexity level A and you want to move to complexity level B, this Atlas will tell you what the products are at the next level enabling you to choose a development path that makes maximum use of your existing workforce and physical infrastructure. This was the genius of the move of Akron into plastics when their rubber industry grew legs and walked. One more example from education.

The Economic Policy Institute has published a study by Stanford Professor Martin Carnoy, et. al. titled, "Bringing it Back Home: Why State Comparisons Are More Useful Than International Comparisons for Improving U.S. Education Policy" (Carnoy, Garcia et al. 2015). Education in America is another example of "American Exceptionalism" (Lipset 1996). Whereas most countries have one national school system, in America there are 51 state systems and 13,000 local school districts. Carnoy argues that instead of being riveted by international comparisons, American policy makers would be better served by disaggregating national data and analyzing the differences among states, particularly neighboring states where performances differ markedly. His thought is that neighboring states are more likely to share characteristics that will make policy innovations in one state relevant to the circumstances in the other. Korean parents pay many thousands of dollar for students to cram for national exams. Finland has a nationwide, five-year preschool program and a homogeneous, relatively high income population. Such national practices are not relevant to the circumstances in the US with its wide range of diverse student sub-groups and diverse regions. It would be much more helpful to Connecticut to know what the policies were that neighboring Massachusetts adopted that has enabled Massachusetts to so consistently outpace Connecticut in student gains over the past seven years.

Adjacency has dimensionality that is tied to the deep structure of social space. The Haitian boys felt like fish out of water in the Brooklyn school. Joining a community of other Haitian boys gave them "nearest neighbors" in an evolutionary sense, creating a connectivity that enabled them to make positive developmental choices.

The definition, nature, and dynamics of space has been an issue for quite some time. In 1950 Francois Perroux pointed out that the grasp of space as an abstract phenomenon had not yet permeated economics. It could very well be that space and economics are orthogonal concepts that require thorough and separate explication before they can be successfully combined. Perroux warns, "We shall risk losing the construction of a new world and a new economy if we persist in thinking of them in terms borrowed from the old world and the old economy. " (Perroux 1950) He speaks of the need to dissociate "economic space" from "human space" and proceeds to lay out the dimensions along which economic space might usefully be defined. "Economic Space: Theory and Applications" an absolutely fascinating piece and could very well provide a model, or at least a starting point for the explication of "social space".

There are two things we know about this space: we know what adjacency means, and we know what networks are. These two characteristics determine what can happen in the space in which the complex system is embedded. What we do not know is whether the space itself has a context (like gravity for the cosmos) and if so, what the nature of that context might be. Further we do not know how the dimensionality of the system's adjacency affects its behavior. Do these dimensions combine in interesting ways? These are the questions of the topology of social space which is a major component of the research program presented in the Conclusion where three tool are highlighted: network analysis, the adjacency matrix; and fitness landscapes. The first two are familiar but the latter might require some new learning. Also, the pressure for change is not an isolated phenomenon. Many disciplines are feeling the same pressure and opportunities for collaboration abound in the areas of sharing mathematical models, sharing data, and sharing problem definitions (imagine this pumping station is a heart muscle, for example). Social scientists need to look to their own rich history; resurrect and reexamine some of their own problem definitions, datasets, and analyses and retrofit them to the new understanding of social dynamics. If this is done from inside a framework established to accommodate the analysis of complexity, as a beginning strategy, this might help us build our intuition about what operating from inside such a framework means.

Complexity Science is conceptually huge, but we have some guideposts going forward. I believe these are the self-similarity of nature (e.g. the inverse square law), nature's preference for certain modus operandi (e.g. clusters, orbits, networks . . .), and the power and consistency of mathematics in revealing both (e.g. the standard equation of particle physics). Witness the fact that network scientists have defined a quantity which they call the "network energy". On this quantity they have further defined the "network Hamiltonian". The Hamiltonian in physics is the total system energy. This mathematical sleight of hand brings together in one concept, the Hamiltonian, unimaginably distant scales -- one at the human scale and beyond (I say beyond because there are networks of telescopes in space), and the other at a scale so small that interactions occur at speeds faster than the speed of light. At this scale, signals can be transmitted galactic distances at speeds that can only be measured as simultaneous -- that is, no time passes between the initiation and the reception of the signal (and given the space-time equivalency of general relativity, that can also be read as there being no space between them!). This is the realm of quantum entanglement. What is important about this is the identity of the mathematical formalisms. We have learned that that implies scaling of a natural phenomenon. This helps to solidify the ground beneath us as we continue to examine the role of networks in shaping space.

In responding to the upheavals in economics, we must be careful not to throw the baby out with the bath water. Brian Arthur cautions against this very thing in discussing the future of economics education. Quantum mechanics is a very different paradigm from Newtonian mechanics, and from the perspective of quantum mechanics, there is much about Newtonian mechanics that is wrong; however, in growing physicists we don't start them out on a diet of quantum mechanics. The pabulum of classical mechanics nourishes them for years and it's not until they have Newtonian mechanics, optics, and electro-magnetism under their belts that they are introduced to solid food. We are in a very exciting period of massive scientific dislocation. It is exciting because just as many citadels are being challenged, so also are many new ideas emerging. However, just as now happens with Newtonian mechanics, the correction of the ways in which the old thinking is incorrect can happen in the context of higher learning as we gradually adjust the pedagogies of inculcating in the discipline's professionals the issues, language, methodologies, and culture of the social sciences.

Further, despite the significant upheavals in economics of late, the science has been extremely useful all these years. Complexity science offers new perspectives, new ways to understand what's been going on, and new avenues for research. For example, Stiglitz has likened both the adjustment of the Great Recession and that of the Great Depression to a phase change in the economy. Now this begs the question of how one defines a phase change in social systems, but Stiglitz' description certainly makes sense. The latter phase change was from an agricultural to an industrial economy, and the former was from an industrial to an information-based economy. Both necessitated structural changes and major adjustments to the size of the labor force. Does that mean that phase diagrams similar to Figure 4 can be drawn for social systems? That would be an amazingly powerful policy tool! What system parameters are involved in the ramp up to a phase change?   In water they are temperature, pressure, and volume. Are we looking for analogs to these parameters in social systems, or something completely different. And do we expect to find only three phases? Remember, a phase change is a radical alteration of the structure or state of the system created by a relatively small change in the value of relevant system parameters, and the system structure is defined by the value of these parameters (e.g. at atmospheric pressure, liquid water becomes solid at zero degrees Celsius). What a potentially fruitful avenue of research!

Proposed Research Framework

Five research directions are proposed.

1. Search Big Data and social science research archives for opportunities to rigorously extend natural laws into the arena of social systems: What is the meaning for social

systems of co-evolution, phase change, . . . ? What are the controlling oscillators? Is it possible to quantify the magnitude and direction of a "social field" and any associated "force"? What does dimensional analysis tell us? Can we meaningfully define a "social Hamiltonian"?

2. Build precisely calibrated materials-based models of social systems to test attribute theory.
3. Conduct further inquiries into the topology of social space: What is the dimensionality of social space; does each dimension have its own topology, how do these dimensions and topologies combine, if at all? If networks capture the topology of socials space, does that not imply that such space has no independent topology but is completely dependent on frame of reference? Are there analogs to this in the natural sciences? Is there a corollary of this that suggests that each interaction has its own topology defined by the network in which the interaction is embedded?
4. Examine the dynamics of adjacency: What is the significance of multidimensional adjacency? Are some adjacencies more important than others?
5. Can spatial econometrics be modified to serve the purposes of the analysis of complexity?

Conclusion

The power of a change in paradigm can get muted by the need that those involved in and committed to the old paradigm feel to keep things as they are. In complexity science terms, this is called "lock-in" Lock-in can also occur when the conceptual barriers to implementing the new paradigm are too high. That is that the changes required can create circumstances that are so radically different from the traditional patterns that the institution undergoing this change is in danger of becoming unrecognizable to its current adherents. Conferences like this can help social scientists avoid acting as barriers to embracing the new paradigm of complexity science; they can help identify places to begin; and they can suggest several paths forward that would add value without unduly destabilizing the field.

A big part of the effort here has been to expand the field's definition of space to include social space, and to place the topology of social space at the center of the field's research agenda. We have centered on the characteristic of adjacency as a defining feature of complex adaptive systems of all sizes and envisioned adjacency a having a key role to play in problem solving in policy analysis. We currently have a kind of "mean field" approach to social space and it is not at all clear that is correct. Because everything clusters, we do not seem to feel the need to question what it is about space that makes that so. At the scale of human communities, it's the opposite problem from looking for high energy particles. It could be that we are unable to perceive the effects on space of social relationships and interactions because the dimensions of our investigations are too crude. Think of scaling in the opposite direction in the search for high energy particles. We build highly tuned accelerator to investigate quantum spaces. What if the relevant dimensions of social space are global and the relevant wavelengths are Leontief? There is an interesting challenge for Big Data! If we want to see these effects, we have to build the right kind of accelerator. It could be something as simple as defining what kinds of networks are relevant, mapping them out, populating them with data, and tracing their interactions. Scaling and self-similarity work in both directions. Physics has quite a head start so the action down-scale is quite well developed. The proposal embodied herein is that the social science disciplines go upscale in a disciplined fashion using what we have learned from complexity science and examine these effects disaggregated and at a larger scale. It could have interesting implications for world peace.

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