

Doomsday

The article "Doomsday" by von Foerster, Mora, and Amiot [*Science* 132, 1291 (1960)], although perhaps written and published with an obvious tongue-in-cheek attitude, has received some publicity in the newspapers, and there is danger that it may be taken too seriously. At least one well-established biological fact has been omitted from the calculations: in human beings it still takes about 270 days from conception to delivery. This fact sets an ultimate limit upon the productivity factor a . If we consider only the reproductive female population (assuming the presence of enough males to maintain the necessary conception rate), it is apparent that the doubling time cannot ever be much less than $\frac{3}{4}$ year. If von Foerster's equation is valid until this doubling time is reached, the curve at this point has to depart from the power function and revert to an exponential,

$$N = e^{-at},$$

where a cannot exceed

$$0.69315/0.75 = 0.925 \text{ yr.}^{-1}$$

From von Foerster's Eq. 12,

$$a = 0.99/\tau,$$

so the power function fails at

$$\tau = 0.99/0.925,$$

or 1.07 years before "dooms-time," when the world population would only be, from von Foerster's Eq. 11,

$$N = 1.79 \times 10^{11}/\tau^{0.99} \\ = 1.67 \times 10^{11},$$

a value which corresponds to a population density less than 5 times that of Japan at present. Of course, males and children add something to the problem, but 1.7×10^{11} is far short of infinity, so there is still a ray of hope.

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The essay in doomsdayism of von Foerster, Mora, and Amiot is to be commended. With the exception of their remarks about Malthus, their conclusions are essentially correct. I say

this because essentially these same conclusions can be arrived at from Malthusian principles.

People who are without food and water for any extended time die first of dehydration and then of starvation. Hence one can predict with confidence that food and water supplies, F , will limit human populations (deserts, highway U.S. 66, 110°W, 1960). This idea can be expressed symbolically as follows:

$$dN/dt = k_1 F \quad (1)$$

where N is the population size, t is the time, and k_1 is the rate constant for the conversion of food into people.

Because food supplies are ultimately limited only by available carbon and its rate of conversion into food by solar energy, we can also predict, also confidently, that the limiting rate of food supply is constant. The rate of food accumulation is zero, however, because the limiting population is hungry. It is also certain that in order to reach a limiting population, all surplus food supplies would have to be consumed, or

$$N = N_{\max}, F = 0 \quad (2)$$

Substituting Eq. 2 in Eq. 1 and integrating, we find, sure enough, that the human population becomes maximal; thank goodness it wasn't infinite after all.

It should be carefully noted that the limiting population may turn out to be larger than that estimated above because people may become smaller.

In conclusion, we should not sell Malthus short. His work in theoretical demography is so nearly contemporary as to make one wonder. There is a solution that has not yet been suggested (except by Swift, for one special case): cannibalism.

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In the article "Doomsday," the assumption is made that the fractional rate of growth of population will increase with the population; consequently, as the population becomes larger, the fractional rate of growth becomes larger and before long exceeds the maximum possible rate of increase permitted by the biology of the human species. It seems obvious that such a theory has no relation to reality and is of no value whatever in predicting future populations.

It is possible, however, to use the methods of this article, starting from more plausible assumptions, and to arrive at population growth curves which

not only are in agreement with the facts of the past but which do yield helpful suggestions as to how the population will grow in the future. Such a formulation has been made, in accordance with the ideas of Raymond Pearl reflected in Eq. 2 of von Foerster *et al.*

The basic assumption is that the population increases at a rate which is proportional to the product of the population and another term which is equal to the supportable population of the region minus the population itself at that time, all divided by the supportable population at the same time. This is the same as Pearl's basic differential equation except that he calls the so-called supportable population the ultimate population and treats it as a constant. In the new formulation the supportable population is considered to be a function of time—namely, a constant plus another constant times time. The resulting differential equation is easily solved in general form, and curves have been constructed in terms of general parameters which make it possible and convenient to extend the historical data of population of a given city or region into the future. The assumption that the supportable population increases with time is in agreement with the assumption of von Foerster *et al.*—namely, that science and technology do increase the ability of a region to support its population.

Using these theories and the set of curves that have been constructed, we find that the population of the United States agrees remarkably well with the appropriate curve from the family of curves referred to, starting with census data for 1790 and ending with data for 1960. The simpler logistic curve of Pearl fails to give agreement after 1940. One's prediction of future population of the United States depends of course upon the choice of constants, and this in turn depends upon one's estimate of the rate of increase of the ability of our territory to support the future population. Whether this ability increases linearly with time or at a faster rate seems to me to be a matter of conjecture at this time. In any case, such a formulation does offer promise of assistance to those who wish to predict future populations, and the absurd results reported in the article "Doomsday" should not discourage us from making attempts of this sort.

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The article by von Foerster, Mora, and Amiot would be too ridiculous to comment on if it were not such an out-

standing example of the inadmissible use of mathematics to prop up a manifestly absurd conclusion. I suppose that the authors are aware of that absurdity, although the tone of the article gives little ground for the supposition, but I wonder why they are not also aware that such articles run the very real danger of increasing the mistrust that many have always shown even of the legitimate uses of mathematics.

The article is so easy to criticize on the basis of the too free use of unsupported hypotheses (particularly Eq. 3) that I shall not do so. Instead, I shall show that *even if the stated hypotheses are accepted the conclusion does not follow*.

It is assumed in the article that the "productivity" α of a population with N members is given by

$$\alpha = \alpha_0 N^{1/k}$$

(Eq. 3), where α_0 and k are constants. The authors then use the "fact" that the rate of change of population is given by

$$\begin{aligned} dN/dt &= \alpha N \\ &= \alpha_0 N^{1+1/k} \end{aligned} \quad (1)$$

to conclude that N goes to infinity at some finite value (A.D. 2027) of the time.

I wish only to point out that this nonsense does not arise if one only recalls that the size of a population is always an integer. As a result, the expression dN/dt has no real meaning except as an approximation, a fact the authors do not bother to point out. Eliminating this approximation, we see that Eq. 1 should read

$$N(n) - N(n-1) = \alpha_0 [N(n-1)]^{1+1/k}, \quad n = 1, 2, \dots,$$

where n refers to the generation under consideration and the unit of time has been taken as a generation. Recalling that $N \geq 1$ for all n since N is an integer, we see that

$$\begin{aligned} N(n) &\leq N(n-1) + |\alpha_0| [N(n-1)]^{1+1/k} \\ &\leq (1 + |\alpha_0|) [N(n-1)]^{1+1/k} \end{aligned}$$

Thus,

$$N(n) \leq (1 + |\alpha_0|)^{k[(1+1/k)n-1]} [N(0)]^{k^n}$$

which is clearly finite for all n .

The argument here should not be misconstrued. The point is not that the world's population growth is not a serious problem but only that progress toward resolution of the problem is in no way served by publication of arguments which, on their face, must be false and which may do some incidental harm. The authors express the hope that their article will "add some fuel to the heated controversy about whether or not the

time has come when something has to be done about population growth control." If the article has this effect, it can only be on a controversy among fools.

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We appreciate the opportunity to comment on the remarks which have been made with respect to our article "Doomsday." There are two points which seem to need further clarification. Since we erroneously believed that these points are part of the household furniture of the scientific community, we apologize for having neglected to restate them explicitly. The first refers to the relation between theory and reality, and to the supportability of a hypothesis. We believe that support of a hypothesis is gained through compatibility with experimental observation (1) rather than by arguments about what should be the case or what should not be the case. This compatibility establishes the relation between theory and reality and serves as a touchstone for accepting or rejecting a hypothesis. If some of our readers express doubt whether or not our simple hypothesis (Eq. 3) has any connection with reality, we obviously failed to keep them interested in this subject long enough to turn to our Fig. 1, which offers a comparison between theory and observation. Although we know that such a comparison, however favorable, will never prove the "truth" of a hypothesis, we pointed out that it seems that our Eq. 11 may, at least, "serve as an adequate empirical formula for presenting most of our recorded data on human population growth" (2).

The second point refers to the interpretation of singularities of the form

$$\lim_{x \rightarrow x_0} y = \infty$$

appearing in the description of the behavior of some finite physical systems. Expressions of this form can be found galore. For instance, let x and y represent, respectively, velocity and pressure at Mach 1 (3, pp. 3-118); or voltage and current at breakdown voltage in gaseous conduction (3, pp. 4-171); or wavelength and index of refraction in optical absorption bands (3, pp. 6-63); or temperature and magnetic susceptibility at Curie point in the theory of ferromagnetism (3, pp. 4-118); and so on. Physical theory behind these expressions is termed neither absurd nor ridiculous, nor is it customary to deny that such theories have predictive value because of these singularities. On the

contrary, since the generally accepted interpretation of expressions such as these, in which a parameter increases rapidly beyond all bounds, is that the system as a whole becomes highly unstable in the vicinity of the critical value x_0 of the corresponding parameter, these singularities serve as welcome warning signals that some breakdown of the system's structure is to be expected.

With respect to the first letter, by Robertson, Bond, and Cronkite, we are very happy to note that this medical research team went along so well with our proposed thesis of "adequate technology," because they obviously must have in mind some tricks for reducing the age of puberty in the human female—the greatest bottleneck in speeding up the rate of reproduction. But who are we to argue with doctors about such points of physiology? However, we may argue their mathematics, because (i) they used a wrong equation for calculating dooms-time for a particular doubling time, and (ii) they failed to follow up their own argument by omitting to calculate the population at doomsday according to the proposed exponential. With our expression for doubling time Δt_2 (that is, Eq. 13, and not Eq. 12), one finds the corresponding dooms-time to be $\tau_1 = 2.25 \Delta t_2$, and invoking Eq. 11, one obtains N_1 , the population on that date. With the aid of the suggested exponential we have N_0 , the "finite" population at doomsday:

$$N_0 = N_1 2^{1/\Delta t_2} = 3.70 \cdot 10^{11} / (\Delta t_2)^{0.60}$$

With the suggested value of $\Delta t_2 = 0.75$ one obtains $N_0 = 5 \cdot 10^{12}$. This corresponds to a population density 15 times that of Japan and about 10 percent that of New York City today. We predicted that this population density would occur on 1 January, A.D. 2024, plus or minus 5.5 years. But according to the arguments advanced by Robertson, *et al.*, we will have this squeeze just 1000 days later. If this is considered to be a ray of hope, the ray is very dim indeed.

We share Hutton's admiration for T. R. Malthus, whose omnipresence in the minds of pessimists as well as optimists we believed we had pointed out.

Howland's suggestion for an approach to population problems is formulated in the differential equation

$$\frac{dN}{dt} = \epsilon_0 N (1 - N/N_0)$$

where ϵ_0 is a constant and N_0 is the "supportable population." Although this hypothesis may be plausible, it has unfortunately no relation to reality when

confronted with estimates of the human global population, unless, as Howland points out, *ad hoc* adjustments for N_0 are made as time goes on. Thus, this theory requires development of a theory for N_0 as a function of t or N . No such function, to Howland's and our knowledge, has as yet been suggested which would fit past data over a period longer than, say, ten generations. In this dilemma we would like to propose, in all modesty, to try tentatively the following, perhaps not too implausible, hypothesis—namely, that $N_0(N)$, the supportable population, is almost always somewhat larger than the instantaneous population N . We suggest:

$$N_0 = N / (1 - \frac{\alpha_0}{\epsilon_0} N^k)$$

with the constants α_0/ϵ_0 and k to be determined by observation. We hope that this suggestion meets with Howland's approval, because it catches three flies with one stroke. First, it expresses, in some sense, our principle of "adequate technology," to which Howland has no objections; second, it will enable Howland's proposed differential equation, when properly integrated, to represent human population growth over more than a hundred generations with a mean deviation of less than 7 percent; and, third, it eliminates guesswork about a quantity which is, in principle, inaccessible to experimental observation—namely, N_0 , the size of the supportable population. This is easily seen by inserting our suggested function into Howland's proposed differential equation, which leads, after integration and adjustment of the constants by the method of least squares, to our Eqs. 11, 12, and 13, which are free of unobservable parameters. We hope that with this little excursion we have supplied Howland with precisely that formulation which, according to him, "does offer promise of assistance to those who wish to predict future populations."

Unfortunately, politeness forbids us to respond to Shinbrot's remarks because this would involve him in a controversy which—in his own words—can only be on "a controversy among fools." Otherwise we would have pointed out our agreement with his feeling that it is unkind to perform a Dedekind cut on a man. On the other hand we could not write our differential equation in the form suggested by Shinbrot because we do not know of any integer-triple $N(n), N(n-1)$, and α_0 , which would fit for $k \neq 1/i$ ($i=1,2,3,\dots$), his suggested difference equation. Obviously he must know such triples, and thus his suggested relationship will remain forever "Shinbrot's last theorem."

In the meantime, while we were displaying our wits and know-how in more or less learned discussions about the perennial question of how many angels can dance on a pin point, over ten million real people of flesh and bone, with hopes and desires, with sorrows and pain, have been added to our family of man. Our responsibility demands that we be ready with an answer when these millions ask for their right to live the span of their human condition in dignity.

Let us join forces so that we will not be caught in a dispute seen prophetically by Francesco de Goya y Lucientes: "Of what will they die?"



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References and Notes

1. B. Russell, *Human Knowledge* (Simon and Schuster, New York, 1948), p. 481.
2. For comparison of our Eq. 11 with estimates of the prehistoric human population, we are grateful to F. Meyer for having drawn our attention to his article "L'Accélération de l'évolution," in *L'Encyclopédie Française* (Larousse, Paris, 1959), vol. 20, p. 24.
3. E. V. Condon and H. Odishaw *Handbook of Physics* (McGraw-Hill, New York, 1958).