Remarks 4-5 on the asymptotic results in Wang and Wang (2009)

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The asymptotic results in our paper were obtained by assuming $p = 2$ for simplicity, where p is the dimension of the quantile coefficients with the first element corresponding to the intercept. For general situations, the rate of the bandwidth h_n would depend on p. More specifically, we have the following two remarks.

Remark 4. The consistency of the proposed estimator (Theorem 1) holds for arbitrary p, as condition (1.4) in Chen et al. (2003) only requires the uniform consistency (no specific rate requirement) of the conditional Kaplan-Meier estimates, which holds by Corollary 2.1 of Dabrowska (1989) when $h_n = n^{-\alpha}$ with $0 < \alpha < 1/(p-1)$.

Remark 5. The asymptotic normality theory (Theorem 2) still holds for p moderately large, which is of practical interest when nonparametric smoothing is applied. As a further note, the result of Dabrowska (1989) was stated using the supremum norm, which can also be used for condition (2.4) of Chen et al. (2003, bottom of page 1592), so our asymptotic normality result holds at least for $q \leq 3$, where $q = p - 1$. Below is the argument. Following from Corollary 2.2 (iii) and definition of b_n in Corollary 2.2(i) of Dabrowska (1989), if we use a high-order kernel (i.e., condition B(ii), which allows one to choose the order of the kernel function according to the dimension of the covariates), and take the smoothing parameter $h_n = n^{-\alpha}$ with $\frac{1}{q+4} < \alpha < \frac{1}{q}$, then

$$||F_n - F|| = o\left(\sqrt{\frac{\log(h_n^{-q})}{nh_n^q}}\right),$$

where $||\cdot||$ denotes the supremum norm. To obtain $||F_n - F|| = o(n^{-1/4})$ (condition (2.4) of Chen et al., 2003), with $h_n = n^{-\alpha}$, we want

$$\log(n^{\alpha q}) = o(n^{1/2-\alpha q}).$$

This requires $\frac{1}{q+4} < \alpha < \frac{1}{2q}$. For $q \leq 3$, we can always find such a α.

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