# NONLINEAR PHENOMENA IN WASHINGTON DC – 2018

## Titles and Abstracts of Talks

8:35am **Mark Allen** *(Brigham Young University)*

**Title:** ***A free boundary problem on three-dimensional cones***

**Abstract:** This talk concerns the one-phase free boundary problem on three-dimensional cones. The one-phase problem has a well-known connection to the study of area-minimizing surfaces, and a result by Morgan classifies when an area minimizing surface is allowed to pass through the vertex of the cone. In this talk we present a result analogous to that of Morgan's. We consider cones depending on a parameter c and study when the free boundary is allowed to pass through the vertex of the cone. When the cone is three-dimensional and c is large enough, the free boundary avoids the vertex. We also show that when c is small enough but still positive, the free boundary is allowed to pass through the vertex. This establishes 3 as the critical dimension for which the free boundary may pass through the vertex of a right circular cone.

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9:25am **Cristi Guevara** *(Louisiana State University)*

**Title:** ***General aspects of the Exciton-Polariton system***

**Abstract:** Exciton-polaritons are half-light, half-matter quantum quasiparticles arising from a strong coupling of quantum wells (excitons) and photon cavities (photons). The quasiparticles emerging from these mixing obey the Bose-Einstein statistics, condensate at high temperatures due to their ultra-small mass; their interaction is described by a quantum-mechanical system, the photon field provides dispersion and the exciton is responsible for a nonlinear behavior similar to the cubic nonlinear Schrödinger (NLS) and Gross-Pitaevskii (GP) equations. As an effort to gain some intuition of the Exciton-Polariton system solutions, we compare them with the NLS. In this talk, I will discuss similarities and challenges of the above systems.

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10:00am **Luiz Farah** *(Federal University of Minais Gerais, Belo Horizonte, Brazil)*

**Title:** ***On the supercritical gKdV equation***

**Abstract:** In this talk we discuss some results for the supercritical gKdV equation, such as well-posedness, existence of maximizers for Airy-Strichartz inequalities, concentration of blow-up solutions and scattering. These results were obtained in collaboration with Ademir Pastor (UNICAMP-Brazil), Brian Pigott (Wofford College-USA), Felipe Linares (IMPA-Brazil), Henrique Versieux (UFMG-Brazil), Nicola Visciglia (UNIPI-Italy). The author was partially supported by CNPq-Brazil and FAPEMIG-Brazil.

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11:15am **Jacob Bedrossian** (University of Maryland)

**Title:** ***Landau damping and nonlinear echoes***

**Abstract:** In this talk we will discuss some of the intricacies of Landau damping in the collisionless Vlasov equations or the collisionless limits of Vlasov-Fokker-Planck equations. We will discuss the construction of solutions to the Vlasov-Poisson equations on S x R which are arbitrarily close to homogeneous equilibrium in Sobolev regularity but which display arbitrarily long sequences of nonlinear oscillations known as plasma echoes. In particular, these oscillations show that the collisionless linearization is not valid for long times in Sobolev regularity. Further, we show that the inclusion of weak collisional effects suppress these plasma echoes and make it possible to obtain Sobolev regularity results. We also prove that Debye shielding and dispersive effects can suppress such nonlinear oscillations (joint with Nader Masmoudi and Clement Mouhot). Combined with the existing infinite regularity results of Mouhot and Villani, these results together confirm and refute a variety of conjectures made by both mathematicians and physicists over the years regarding Landau damping near homogeneous equilibrium.

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1:00pm **Justin Holmer** (Brown University and NSF) – Joint with Colloquium

**Title:** ***Blow up for the critical Zakharov-Kuznetsov equation***

**Abstract:** The generalized Zakharov-Kuznetsov (ZK) equation is a higher dimensional version of the generalized Korteweg-de Vries (gKdV) equation. While the KdV equation and its generalizations have long been studied, a question about existence of blow-up solutions in gKdV has posed challenges and is far from being answered. One of the main obstacles is that unlike other dispersive models (such as the nonlinear Schrödinger or wave equations), the gKdV equation does not have a suitable variance quantity, which is the key to showing existence of finite time singularities. Only at the dawn of this century the works of Martel and Merle established the existence of finite-time blow-up solutions in the critical gKdV equation. In this talk we discuss the existence of finite time blow-up solutions in the corresponding two-dimensional critical ZK equation. We prove sharp pointwise decay estimates for the linear part of ZK, give applications to linearized ZK solutions and some applications to well-posedness and soliton stability problems. This is a joint work with Luiz Farah (UFMG, Brazil), Svetlana Roudenko (GWU) and Kai Yang (GWU).

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2:00pm **Benjamin Dodson** (John Hopkins University)

**Title:** ***A subthreshold scattering result for the cubic nonlinear Schrodinger equation***

**Abstract:** In this talk we discuss a recent result by myself and Jason Murphy. We prove scattering for the focusing, cubic nonlinear Schrodinger equation with data below the soliton threshold. This result was already proved by Duyckaerts, Holmer, and Roudenko. Here ,we do not use concentration compactness, and thus obtain an explicit bound on the scattering size.

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2:50pm **Anudeep Kumar Arora** (GWU, graduate student)

**Title:** ***Scattering in the generalized Hartree equation***

**Abstract:** We consider the focusing generalized Hartree equation in the mass-supercritical and energy-subcritical setting. The characterization of solutions behavior under the so called mass-energy threshold is known for the NLS case from the works of Holmer and Roudenko (radial) and Duyckaerts, Holmer and Roudenko (nonradial) and generalizations (Guevara and others). The scattering behavior is typically proved following the road map developed by Kenig and Merle in 2006, using the concentration compactness and rigidity technique, which is a standard tool by now in the dispersive problems.

In this talk we discuss an alternative proof of scattering in the Hartree case, following the approach of Dodson and Murphy for the focusing 3d cubic NLS equation, which relies on the scattering criterion of Tao, combined with the radial Sobolev and Morawetz-type estimates.

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3:10pm **Jonas Lührmann** (John Hopkins University)

**Title:** ***Probabilistic local well-posedness and scattering for the 4D cubic NLS***

**Abstract:** We consider the Cauchy problem for the cubic nonlinear Schrödinger equation (NLS) in four space dimensions. It is known that for initial data at energy regularity, the solutions exist globally in time and scatter. However, the problem is ill-posed for initial data at super-critical regularity, i.e., for regularities below the energy regularity. In this talk we study the super-critical data regime for this Cauchy problem from a probabilistic point of view, using a randomization procedure that is based on a unit-scale decomposition of frequency space. In the first part of the talk we will explain how the problem of establishing almost sure local existence for the cubic NLS for such random data has some features in common with proving local existence for a derivative NLS equation. Our method is inspired by the local smoothing estimates and functional frameworks from the Schrodinger maps literature. If time permits, we will also turn to the long-time dynamics of the solutions and present some (conditional) almost sure scattering results.
This is joint work with Ben Dodson (John Hopkins) and Dana Mendelson (Univ. of Chicago).

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4:15pm **Xiang Xu** (Old Dominion University)

**Title:** ***A variational approach to Poisson-Nernst-Planck equations***

**Abstract:** The Poisson−Nernst−Planck system of equations used to model ionic transport is interpreted as a gradient flow for the Wasserstein distance and a free energy in the space of probability measures with finite second moment. A variational scheme is then set up and is the starting point of the construction of global weak solutions in a unified framework for the cases of both linear and nonlinear diffusion. The proof of the main results relies on the derivation of additional estimates based on the recently developed flow interchange technique.

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4:55pm **Kai Yang** (GWU, graduate student)

**Title:** ***Stable blow up in critical and supercritical NLS and Hartree equations***

**Abstract:** We discuss stable blow-up dynamics in the L^2-critical nonlinear Schrödinger equation (NLS) equation, proving the log-log blow-up behavior of solutions up to the dimension 12 via the Spectral property. Then, in the L^2-supercritical NLS, we exhibit profiles and the square root blow-up rate in the stable blow-up formations. We next study the generalized Hartree equation with the nonlocal (convolution type) nonlinearity and show that the stable blow-up behavior is similar to the NLS case, thus, concluding that the form of the nonlinearity does not influence the formation of singularities in the NLS-type equations.

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5:10pm **Chong Wang** (GWU, graduate student)

**Title:** ***Analysis and Modeling of Self-organized Systems with Nonlocal Interactions***

**Abstract:** Energy-driven pattern formation induced by competing short and long range interaction is common in many biological and physical systems. We report on our work through two models. The sharp interface model is a nonlocal and non-convex geometric variational problem. The admissible class of the energy functional is a collection of sets where each set is of finite perimeter. The original problem is recast as a variational problem on a Hilbert space through introducing internal variables. We prove the existence of the core-shell assemblies and the existence of the disc assemblies as the stationary points of the energy functional in ternary systems. We also prove the existence of a triple bubble in a quaternary system. The other model is the diffuse interface model concerning minimizers of the Ginzburg-Landau free energy supplemented with long range interaction in inhibitory systems. As model parameters vary, a large number of morphological phases appear as stable stationary states. One open question related to the polarity direction of double-bubble assemblies is answered numerically. More importantly, it is shown that the average size of bubbles in a single-bubble assembly does not depend on the ratio of volume fractions but rather is determined by the long range interaction coefficients and the sum of the minority constituent volumes. In double-bubble assemblies, a two-thirds power law between the number of double bubbles and the long range interaction coefficients in the strong segregation regime is justified both numerically and theoretically. A range of parameters is identified that yields double-bubble assemblies. These two models can be connected through gamma convergence.

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5:30pm **Yanxiang Zhao** (GWU)

**Title:** ***A New Phase-Field Approach to Variational Implicit Solvation of Charged Molecules with the Coulomb-Field Approximation***

**Abstract:** We construct a new phase-field model for the solvation of charged molecules with a variational implicit solvent. Our phase-field free-energy functional includes the surface energy, solute-solvent van der Waals dispersion energy, and electrostatic interaction energy that is described by the Coulomb-field approximation, all coupled together self-consistently through a phase field. By introducing a new phase-field term in the description of the solute-solvent van der Waals and electrostatic interactions, we can keep the phase-field values closer to those describing the solute and solvent regions, respectively, making it more accurate in the free-energy estimate. We first prove that our phase-field functionals Γ-converge to the corresponding sharp-interface limit. We then develop and implement an efficient and stable numerical method to solve the resulting gradient-flow equation to obtain equilibrium conformations and their associated free energies of the underlying charged molecular system. Our numerical method combines a linear splitting scheme, spectral discretization, and exponential time differencing Runge-Kutta approximations. Applications to the solvation of single ions and a two-plate system demonstrate that our new phase-field implementation improves the previous ones by achieving the localization of the system forces near the solute-solvent interface and maintaining more robustly the desirable hyperbolic tangent profile for even larger interfacial width. This work provides a scheme to resolve the possible unphysical feature of negative values in the phase-field function found in the previous phase-field modeling (H. Sun et al, J. Chem. Phys, 2015) of charged molecules with the Poisson--Boltzmann equation for the electrostatic interaction.

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