THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

**SCHOOL OF ENGINEERING AND APPLIED SCIENCE**
**DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING**
**ECE 2110: CIRCUIT THEORY LABORATORY**

# Experiment #10: *Passive Filter Design*

## EQUIPMENT

| *Lab Equipment* | *Equipment Description* |
|---|---|
| (1) Function Generator | Agilent 33522A Function/Arbitrary Waveform Generator |
| (1) Digital Multimeter (DMM) | Agilent 34460A (DMM) |
| (1) Digital Oscilloscope | Keysight DSOX1024G Digital Oscilloscope |
| (1) Breadboard | Prototype Breadboard |
| (1) BNC T-Connector | One input to two output BNC connector |
| (1) Test Leads | Banana to Alligator Lead Set |
| (2) Test Leads | BNC to Mini-Grabber Lead Set |
| (1) BNC Cable | BNC to BNC Cable |

**Table 1 – Equipment List**

## COMPONENTS

| *Type* | *Value* | *Symbol Name* | *Multisim Part* | *Description* |
|---|---|---|---|---|
| Resistor | 3.3kΩ | R | Basic/Resistor | --- |
| Resistor | 510Ω | $R_2$ | Basic/Resistor | --- |
| Capacitor | 820pF | C | Basic/Capacitor | Ceramic Disk, 821J |
| Inductor | 4.7mH | L | Basic/Inductor | --- |

**Table 2 – Component List**

## OBJECTIVES

- Find the frequency response of a series RC and RL circuit
- Plot the magnitude and phase response of a series RC and RL circuit
- Design, build, and test a low-pass filter
- Design, build, and test a high-pass filter
- Find the frequency response of a series and parallel resonance circuit
- Plot the magnitude and phase response of a series and parallel resonance circuit • Design, build, and test a band-pass filter

## INTRODUCTION

This lab will focus on understanding the behavior of common filters and how we can create filters with simple passive components such as capacitors, inductors, and resistors.

### Filters

An electric filter modifies the frequency content of a signal. **Figure 1** shows the four main types of filters: **high-pass** (HPF), **low-pass** (LPF), **band-pass** (BPF), and **band-stop** (notch). A low-pass filter allows low frequencies to pass to the load while attenuating high frequencies. A high-pass filter allows high frequencies to pass while attenuating low frequencies. A band-pass filter allows a range of frequencies to pass while attenuating frequencies outside of that range. A band-stop filter attenuates a range of frequencies while passing frequencies outside of that range.

In **Figure 1**, the x-axes represent frequency (ω) in radians per second. By convention, frequency is represented by the variable ω when its units are radians per second and f when its units are Hertz. The yaxes represent the gain of each filter. In this instance, gain is defined as the voltage across the load divided by the input voltage. As is shown in the figure, the gain of a filter is different at different frequencies.

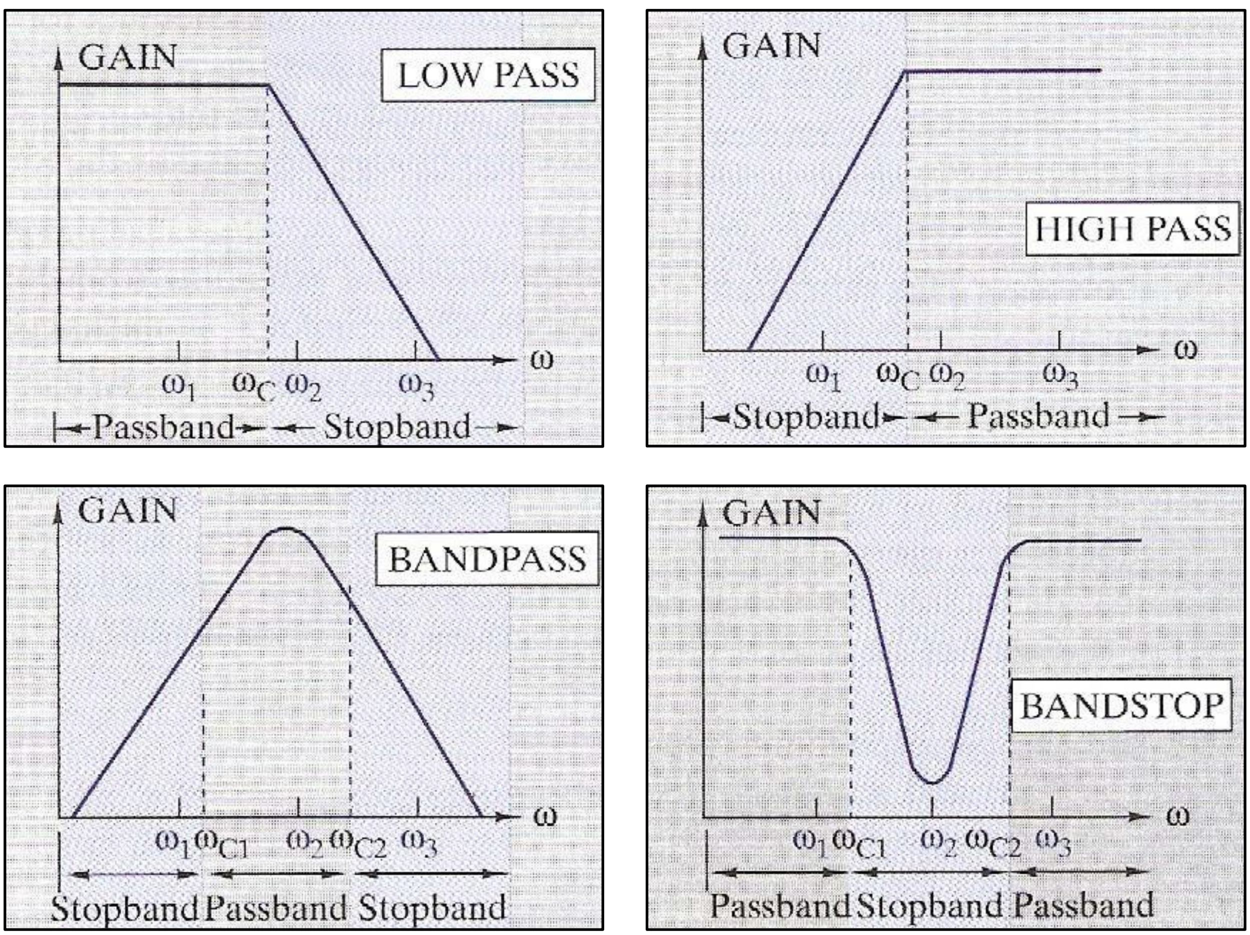


**Figure 1 – Gain Responses (Thomas et al., page 602)**

### *Common Filter Terms*

The range of frequencies that are attenuated is called the **stopband**. The range of frequencies that pass to the load is called the **passband**. The **cutoff frequency** ($\omega_c$ or $f_c$) is the frequency at the transition between the stopband and passband (band-pass and band-stop filters will have two cutoff frequencies). An **ideal filter** passes frequencies in the passband without modifying their magnitude (Gain = 1) and completely attenuates frequencies in the stopband (Gain = 0). However, **ideal filters do not exist in practice**. One convention is to define cutoff frequencies as the frequency at which the magnitude of the voltage at the load is decreased by **3dB** from its maximum value $\frac{V_{max}}{\sqrt{2}}$, called the **-3dB frequency**. There are other ways to define the cutoff frequency, so when reading or specifying $\omega_c$, make sure that you understand which definition is being used. With respect to a filter, a **decibel** (dB) is defined as ten times the logarithm to base 10 of the ratio of the output power to the input power. When the input and output powers are delivered to an equal resistance, a decibel can be defined with respect to the voltage gain of the filter. This derivation is shown in **Equation 1**. Using this definition, it can be shown that a 3dB reduction in voltage is approximately equal to a reduction of $\frac{1}{\sqrt{2}}$ in voltage or a reduction of **half the power**.

$$\#\ \boldsymbol{of\ dB} = 10\log_{10}\frac{Pout}{Pin} = 10\log_{10}\frac{\frac{V_{out}^2}{R}}{\frac{V_{in}^2}{R}} = 10\log_{10}\frac{Vout}{Vin}\frac{Vout}{Vin}\ 10\log_{10}\frac{Vout}{Vin} + 10\log_{10}\frac{Vout}{Vin} = \mathbf{20\log_{10}\frac{Vout}{Vin}}$$

***Equation 1 – Decibel Derivation (Thomas et al., page 603)***

The **center frequency** ($\omega_0$ or $f_0$) is the frequency where the voltage at the load is at its **maximum** value. The **bandwidth** (B) of a filter is the difference between the two cutoff frequencies. The **quality factor** (Q) is the ratio of the center frequency to the bandwidth ($Q = \frac{\omega_0}{B}$). The **gain function** of a filter is the ratio of the magnitude of the frequency response of the filter at the load to the magnitude of the frequency response of the source.

***Note:*** *For passive circuits, the gain must be* ***less than one****.*

As an example, the magnitude and phase of the voltage at the load of a series RL circuit, given a source voltage of 1∠0°V, is shown in Figure P1. The red line illustrates the voltage across the inductor, and the blue line is the voltage across the resistor (load). The magnitude plot shows that the circuit has a cutoff frequency (-3dB frequency) of approximately 30kHz, a passband from 0 to 30kHz, and a stopband from 30kHz to Infinity.

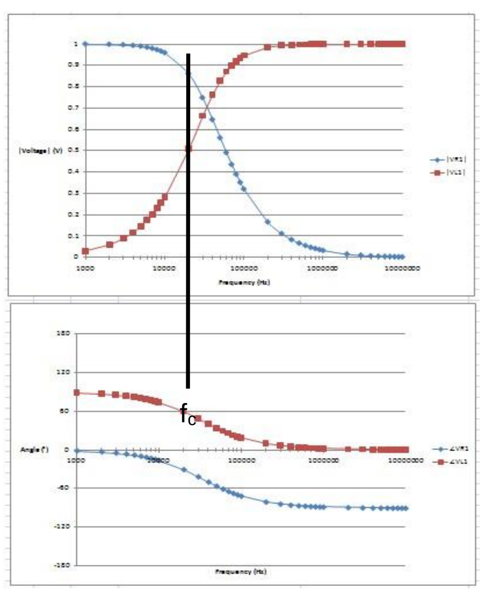


**Figure P1: Low-Pass Filter - magnitude (top), phase (bottom)**

## PRELAB

*Review the provided Excel spreadsheet (lab10_example.xlsx). It demonstrates a partial solution to Part I of the prelab. Using this spreadsheet as an example, produce similar spreadsheets for Parts II-IV.*

**Part I – Series RC Circuit**

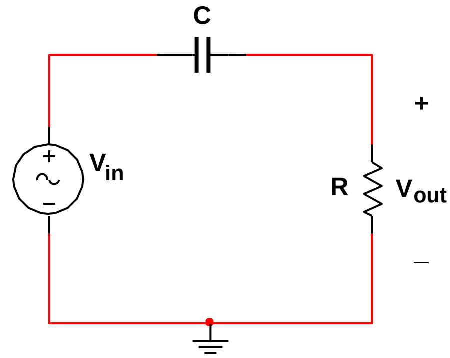


**Figure P.1 – Series RC Circuit**

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_C}$ and $\mathbf{V_R}$ associated with C and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_C}$, $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_C + V_R = V_{in}$ for all frequencies using Excel.

   ***Hint:*** *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*

   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **C = 820pF**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

**Part II – Series RL Circuit**

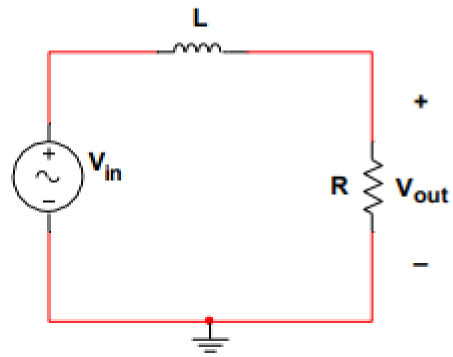


**Figure P.2 – Series RL Circuit**

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_L}$ and $\mathbf{V_R}$ associated with L and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_L}$, $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_L + V_R = V_{in}$ for all frequencies using Excel.

   ***Hint:*** *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

**Part III – Series Resonant Circuit**

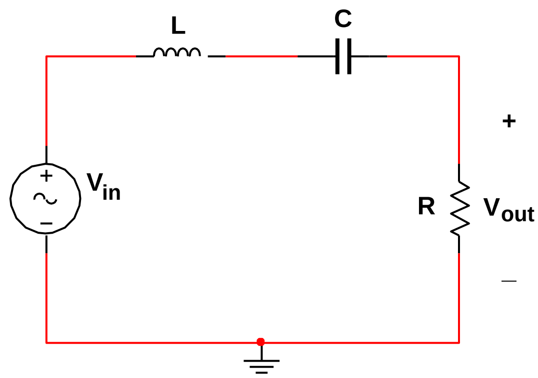


**Figure P.3 – Series Resonant Circuit**

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$ associated with L, C, and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_L + V_C + V_R = V_{in}$ for all frequencies using Excel. ***Hint:*** *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **C = 820pF**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

## Part IV – Parallel Resonant Circuit

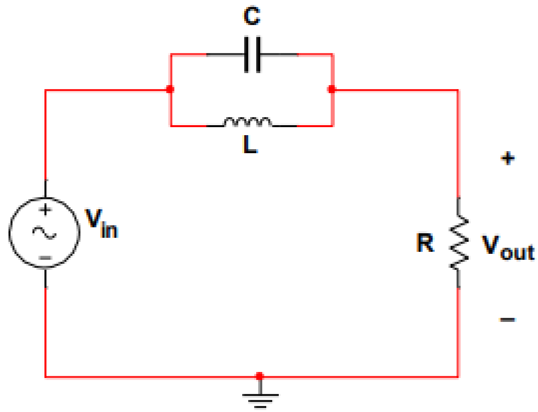


**Figure P.4 – Parallel Resonant Circuit**

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$ associated with L, C, and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that ($V_L$ or $V_C$) + $V_R$ = $V_{in}$ for all frequencies using Excel. ***Hint:*** *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **C = 820pF • R = 510Ω**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

 

## LAB

### Part I – Series RC Circuit Measurement

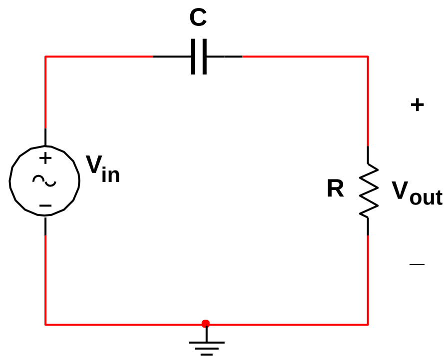


**Figure 1.1 – Series RC Circuit**

1. **Build** the circuit in **Figure 1.1** on a breadboard using the following components:
   - **$V_{in}$ = 1$V_{pk}$**
   - **C = 820pF**
   - **R = 3.3kΩ**
2. **Measure $V_{out}$** (magnitude and phase) for different frequencies from 1kHz to 10MHz.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.

| ***Frequency*** | ***Magnitude*** | ***Phase*** |
|---|---|---|
| **1kHz** | | |
| **10kHz** | | |
| **20kHz** | | |
| **30kHz** | | |
| **40kHz** | | |
| **50kHz** | | |
| **60kHz** | | |
| **70kHz** | | |
| **80kHz** | | |
| **90kHz** | | |
| **100kHz** | | |
| **300kHz** | | |
| **1MHz** | | |
| **5MHz** | | |
| **10MHz** | | |

**Table 1.1 – Series RC Circuit Data**

### Part II – Series RL Circuit Measurement

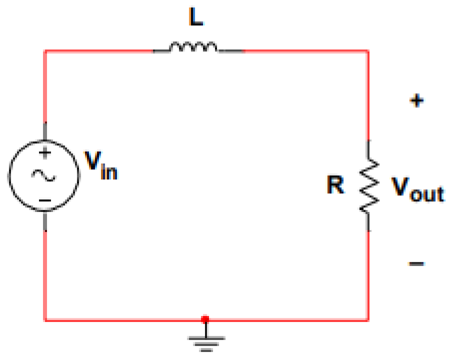


**Figure 2.1 – Series RL Circuit**

1. **Build** the circuit in **Figure 2.1** on a breadboard using the following components:
   - **$V_{in}$ = 1$V_{pk}$**
   - **L = 4.7mH**
   - **R = 3.3kΩ**
2. **Measure $V_{out}$** (magnitude and phase) for different frequencies from 1kHz to 10MHz.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency. 4. **Plot** the **phase difference versus frequency** in Excel using your collected data.

| *Frequency* | *Magnitude* | *Phase* |
|---:|---|---|
| **1kHz** | | |
| **10kHz** | | |
| **20kHz** | | |
| **40kHz** | | |
| **60kHz** | | |
| **80kHz** | | |
| **100kHz** | | |
| **200kHz** | | |
| **300kHz** | | |
| **500kHz** | | |
| **600kHz** | | |
| **800kHz** | | |
| **1MHz** | | |
| **5MHz** | | |
| **10MHz** | | |

**Table 2.1 – Series RL Circuit Data**

## Part III – Series Resonant Circuit Measurement

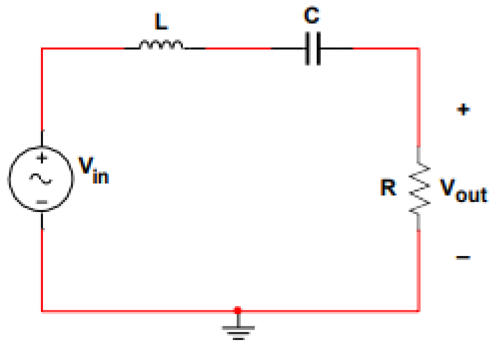


**Figure 3.1 – Series Resonant Circuit**

1. **Build** the circuit in **Figure 3.1** on a breadboard using the following components:
   - $\mathbf{V_{in} = 1V_{pk}}$
   - **L = 4.7mH**
   - **C = 820pF**
   - **R = 3.3kΩ**
2. **Measure** $\mathbf{V_{out}}$ (magnitude and phase) for different frequencies from 1kHz to 10MHz.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.
5. **Find** the cutoff frequencies ($\boldsymbol{\omega_{C1}}$ **and** $\boldsymbol{\omega_{C2}}$), Bandwidth (**B**), center frequency ($\boldsymbol{\omega_0}$) and Quality factor (**Q**).

| ***Frequency*** | ***Magnitude*** | ***Phase*** |
|---:|---|---|
| **1kHz** | | |
| **5kHz** | | |
| **10kHz** | | |
| **20kHz** | | |
| **40kHz** | | |
| **70kHz** | | |
| **80kHz** | | |
| **90kHz** | | |
| **100kHz** | | |
| **200kHz** | | |
| **400kHz** | | |
| **500kHz** | | |
| **1MHz** | | |
| **5MHz** | | |
| **10MHz** | | |

**Table 3.1 – Series Resonant Circuit Data**

**Part IV – Series Parallel Circuit Measurement**

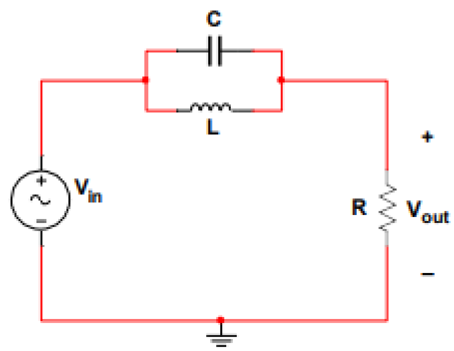


**Figure 4.1 – Parallel Resonant Circuit**

1. **Build** the circuit in **Figure 4.1** on a breadboard using the following components:
    - **$V_{in}$ = 1$V_{pk}$**
    - **L = 4.7mH**
    - **C = 820pF**
    - **R = 510Ω**
2. **Measure $V_{out}$** (magnitude and phase) for different frequencies from 1kHz to 10MHz.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.
5. **Find** the cutoff frequencies (**$\omega_{C1}$ and $\omega_{C2}$**), Bandwidth (**B**), center frequency (**$\omega_0$**) and Quality factor (**Q**).

| ***Frequency*** | ***Magnitude*** | ***Phase*** |
|---:|---|---|
| **1kHz** | | |
| **5kHz** | | |
| **10kHz** | | |
| **30kHz** | | |
| **40kHz** | | |
| **60kHz** | | |
| **80kHz** | | |
| **90kHz** | | |
| **100kHz** | | |
| **200kHz** | | |
| **100kHz** | | |
| **500kHz** | | |
| **1MHz** | | |
| **5MHz** | | |
| **10MHz** | | |

**Table 4.1 – Series Parallel Circuit Data**

**Part V – Low-Pass Filter Design**

1. Use Multisim to **design** and **simulate** a **high-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - -3dB Frequency: 500Hz
    - Tolerances: 5%
2. **Build**, **test**, and **demonstrate** this circuit to the GTA.

**Part VI – High-Pass Filter Design**

1. Use Multisim to **design** and **simulate** a **high-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - -3dB Frequency: 20kHz
    - Tolerances: 5%
2. **Build**, **test**, and **demonstrate** this circuit to the GTA.

**Part VII – Band-Pass Filter Design**

1. Use Multisim to **design** and **simulate** a **band-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - Quality Factor: 1
    - Bandwidth: 15kHz
    - Tolerances: 5%
2. **Build**, **test**, and **demonstrate** this circuit to the GTA.

## POST-LAB ANALYSIS

1. **Compare** the calculated results to the measured results and explain any and all differences.
2. **Describe** the motivation behind defining the cutoff frequency at the point where the gain is -3dB as opposed to -4dB or -5dB.
3. **What type of filter** would you want to implement if you observed **high frequency noise** in your voltage signal?
4. **Describe** a situation where a band-pass filter would be desired.
5. **Does** it make sense to define the bandwidth of a high-pass filter? **Explain**.
6. **Describe** the relationship between $\omega_C$ and $f_C$. Be sure to include the **mathematical relationship**.
7. **Hum noise** is a common phenomenon in electronic devices especially hi-fi equipment. The noise comes from the line (110 Vacrms @ 60 Hz). Using the information you have learned so far, how could you eliminate this noise?
8. **Describe** how the quality factor (Q) is used to distinguish between narrow-band and wide-band filters.

## REFERENCES

[1] Thomas, Roland E., Albert J. Rosa, and Gregory J. Toussaint. *The Analysis and Design of Linear Circuits*. 7th ed. Hoboken, NJ: Wiley, 2012.