THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

SCHOOL OF ENGINEERING AND APPLIED SCIENCE
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
ECE 2110: CIRCUIT THEORY LABORATORY

Experiment #5:
Thévenin's Theorem, Mesh Current, and Node Voltage Analysis

EQUIPMENT

Lab Equipment	*Equipment Description*
(1) DC Power Supply	Keysight E36311A Triple Output DC Power Supply
(1) Digital Multimeter (DMM)	Agilent 34460A (DMM)
(1) Breadboard	Prototype Breadboard
(3) Test Leads	Banana to Alligator Lead Set

Table 1 – Equipment List

COMPONENTS

Type	*Value*	*Symbol Name*	*Multisim Part*	*Description*
Resistor	750Ω	R_1	Basic/Resistor	---
Resistor	1kΩ	R_2	Basic/Resistor	---
Resistor	1.5kΩ	R_3	Basic/Resistor	---
Resistor	3kΩ	R_4	Basic/Resistor	---
Resistor	4kΩ	R_5	Basic/Resistor	---
Resistor	10kΩ	R_6	Basic/Resistor	---
Resistor	12kΩ	R_7	Basic/Resistor	---
Resistor	15kΩ	R_8	Basic/Resistor	---

Table 2 – Component List

OBJECTIVES

- To understand Thévenin's theorem of equivalent circuits
- To analyze and reduce three DC circuits to their Thévenin equivalent circuits by hand
- To analyze and reduce three DC circuits to their Thévenin equivalent circuits with Multisim
- To analyze and reduce three DC circuits to their Thévenin equivalent circuits in la

INTRODUCTION

In this lab, you must find the Thévenin and Norton equivalents for three separate circuits. You will first perform the analysis by hand using the techniques that you have learned in lecture and in your homework. Then, you will simulate the circuits in Multisim, using it to find the Thévenin and Norton equivalents. Finally, you will build the three circuits in lab and measure them to find the Thévenin and Norton equivalents. In your lab report, you will compare and analyze the results from each technique.

Thévenin's Theorem

Thévenin's Theorem for DC circuits states that any two port linear network may be replaced by a single voltage source with an appropriate internal resistance. The Thévenin equivalent will produce the same load current and voltage as the original circuit to any load. Consequently, if many different loads or subcircuits are under consideration, using a Thévenin equivalent usually proves to be a much quicker method of analysis. Any complex circuit consisting of multiple sources and components can be reduced to a single voltage source, the Thévenin voltage, in series with its internal resistance, the Thévenin resistance.

The **Thévenin voltage**, V_{TH}, is found by determining the open-circuit output voltage. The **Thévenin resistance**, R_{TH}, is found by replacing any DC sources with their internal resistances and determining the resulting combined resistance as seen from the two ports using standard series-parallel analysis techniques. A voltage source would be replaced by a short, zero resistance, and a current source would be replaced by an open, infinite resistance. In the laboratory, the Thévenin resistance may be found using an ohmmeter (again, replacing the sources with their internal resistances) or by using the matched load technique. The matched load technique involves replacing the load with a variable resistance and then adjusting it until the load voltage is precisely one-half of the unloaded voltage. This would imply that the other half of the voltage must be dropped across the equivalent Thevenin resistance, and as the Thevenin circuit is a simple series loop then the two resistances must be equal as they have identical currents and voltages.

Norton's Theorem

Norton's Theorem for DC circuits states that any two port linear network may be replaced by a single current source with an appropriate internal resistance in parallel. It is closely related to Thévenin's Theorem, and either simplified representation works equally well. Any complex circuit consisting of multiple sources and components can be reduced to a single current source, the Norton or short-circuit current, in parallel with its internal resistance, which is the same as the Thévenin resistance.

The **Norton current** is found by determining the short-circuit output current, I_{SC}. The internal resistance can be determined in the same manner as was explained above for the Thévenin resistance.

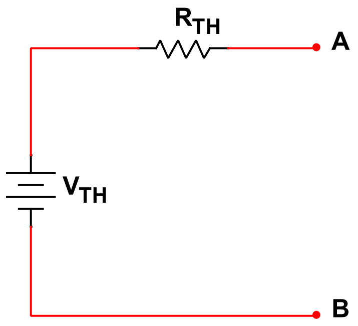

Figure 1 – Thévenin Equivalent Circuit

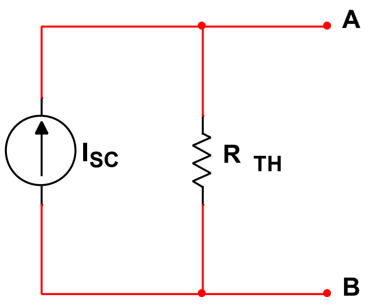

Figure 2 – Norton Equivalent Circuit

Mesh Current Analysis

Multi-source DC circuits may be analyzed using a mesh current technique. The process involves identifying a minimum number of small loops such that every component exists in at least one loop. **Kirchhoff's Voltage Law** is then applied to each loop, meaning that the algebraic sum of the voltages around each loop must equal zero. The loop currents are referred to as mesh currents as each current interlocks or meshes with the surrounding loop currents. As a result, there will be a set of simultaneous equations created, an unknown mesh current for each loop. Once the mesh currents are determined, various branch currents and component voltages may be derived.

Node Voltage Analysis

Multi-source DC circuits may be analyzed using a node voltage technique. The process involves identifying all of the circuit nodes, a node being a point where various branch currents combine. A reference node, usually ground, is included. **Kirchhoff's Current Law** is then applied to each node, meaning that the algebraic sum of the currents into and out of each node must equal zero. Consequently, a set of simultaneous equations are created with an unknown voltage for each node with the exception of the reference. In other words, a circuit with a total of five nodes including the reference will yield four unknown node voltages and four equations. Once the node voltages are determined, various branch currents and component voltages may be derived.

How to Use All Three DC Voltage Outputs

For Part II of this lab, you will need to build a circuit that has three separate voltage sources. It is very important that you know how to properly use all three of the voltage outputs from the DC power supply simultaneously before attempting to do so in lab.

Remember that the KEYSIGHT E36311A Triple Output DC Power Supply has **6V**, **25V**, and **-25V** outputs. Because the positive and negative sources are independent, a common ground is necessary to ensure that there are no "floating" reference points in the circuit. If the negative sides of the voltage sources are grounded in your schematic, they need to be tied together to the same ground in lab. To do this, simply wire the common (black) terminal from each side together to the green **earth ground** terminal in the middle as shown below in **Figure 3**.

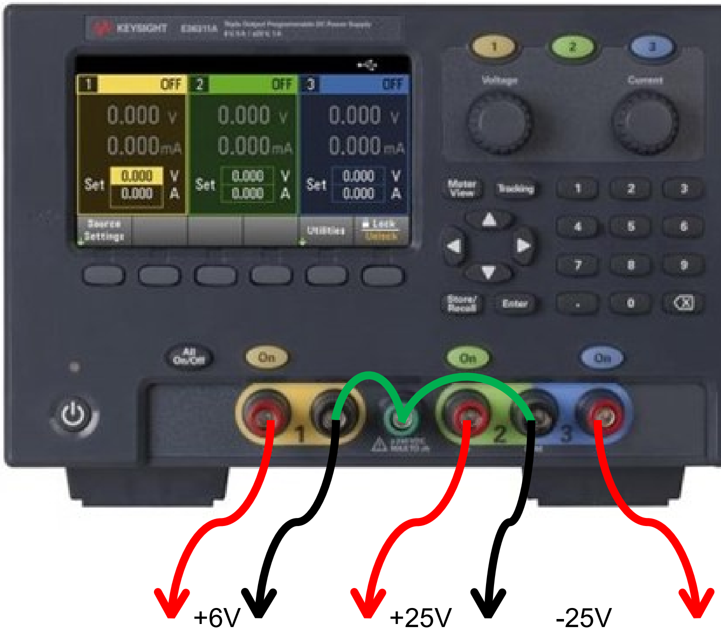

Figure 3 – DC Power Supply Connections

PRELAB

This prelab consists of three circuits to be reduced using Thévenin's Theorem. You are required to complete both hand calculations and Multisim simulations before coming to lab. **This prelab requires more analysis than previous labs; please allocate the proper amount of time to complete it early on.**

Part I – Circuit #1 Analysis

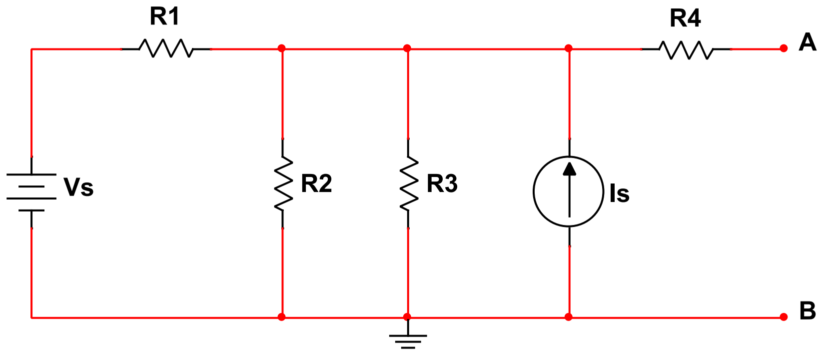

Figure P.1.1 – Circuit #1

In **Figure P.1.1**, the voltage source **Vs = 6V** and the current source **Is = 10 mA**.

1. **Calculate** the following from the perspective of terminals A and B. ***Show all work.***
   a. Thévenin voltage (V_{TH})
   b. Thévenin resistance (R_{TH})
   c. Norton current, also called the short-circuit current (I_{SC})
2. **Use** your results to fill in the appropriate values for **Figure P.1.2** and **Figure P.1.3** below:

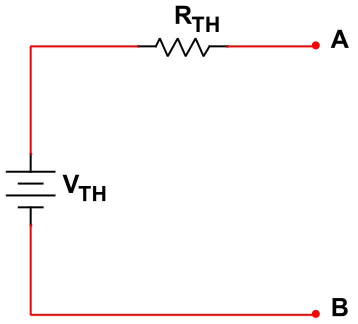

Figure P.1.2 – Thévenin Equivalent of Circuit #1

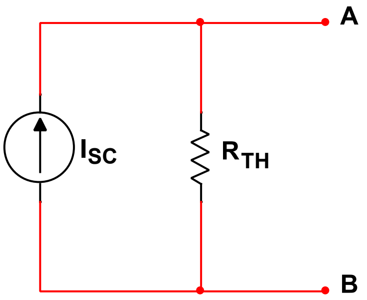

Figure P.1.3 – Norton Equivalent of Circuit #1

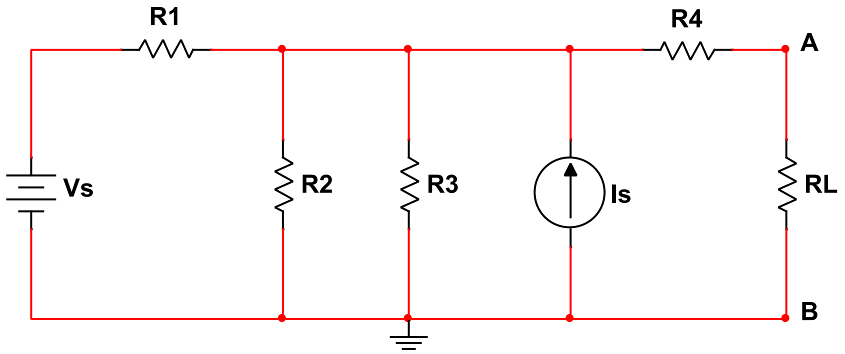

Figure P.1.4 – Circuit #1 with Load Resistor Attached

3. **Calculate** the **voltage across** and **current through R_L** for the circuit in **Figure P.1.4**, which has a 1kΩ load resistor RL attached across terminals A and B.

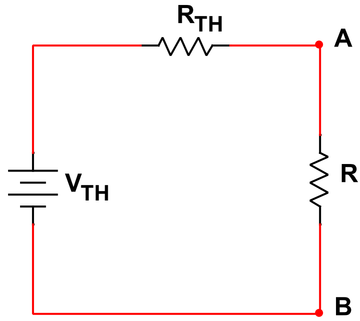

Figure P.1.5 – Thévenin Equivalent with Load Resistor

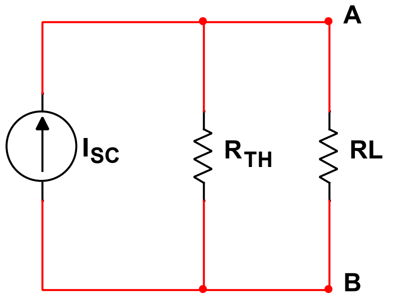

Figure P.1.6 – Norton Equivalent with Load Resistor

4. **Calculate** the **voltage across** and **current through R_L** for these circuits, which have a 1kΩ load resistor RL attached across terminals A and B.
5. **Simulate** the circuit from **Figure P.1.1** in Multisim:
    a. Use the tutorial on the lab website (***Multisim Tutorial #2: Using Multisim to find V_{TH}, I_{SC}, R_{TH}***) to learn how to find V_{TH}, I_{SC}, and R_{TH} for a circuit in Multisim.
    b. **Build** the circuit in **Figure P.1.1** in Multisim and find its V_{TH}, I_{SC}, and R_{TH}.
    c. **Build** the equivalent circuits in **Figure P.1.2** and **Figure P.1.3** in Multisim.
    d. In Multisim, attach a 1kΩ load resistor (R_L) to the circuit as you did in **Figure P.1.4**. **Measure** and **record** the simulated voltage across and current through R_L.
    e. In Multisim, attach 1kΩ load resistors (R_L) to the equivalent circuits in **Figure P.1.5** and **Figure P.1.6**. **Measure** and **record** the simulated voltage across and current through R_L.

Electrical Quantity	***V_{TH}***	***R_{TH}***	***I_{SC}***	***Original Circuit***		***Equivalent Circuit***	
				V_{RL}	***I_{RL}***	***V_{RL}***	***I_{RL}***
Calculated							
Simulated							
Percent Error							

Table P.1 – Circuit #1 Data

Part II – Circuit #2 Analysis

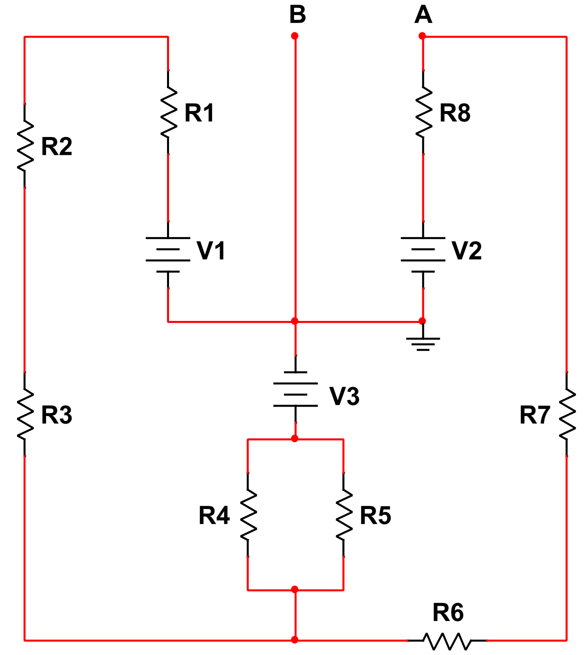

Figure P.2.1 – Circuit #2

In **Figure P.2.1**, the voltage sources **V1 = 3V**, **V2 = 6V**, and **V3 = -6V**.

1. **Calculate** the following from the perspective of terminals A and B. ***Show all work.***
   a. Thévenin voltage (V_{TH})
   b. Thévenin resistance (R_{TH})
   c. Norton current, also called the short-circuit current (I_{SC})
2. **Use** your results to fill in the appropriate values and draw the **Thévenin** and **Norton equivalents** as you did in Part I.
3. **Attach** a 1kΩ load resistor RL across terminals A and B in **Figure P.2.1** and **calculate** the **voltage across** and **current through R_L** for the new circuit.
4. **Attach** a 1kΩ load resistor RL across terminals A and B in the equivalent circuits that you drew and **calculate** the **voltage across** and **current through R_L** for the new circuits.
6. **Simulate** the circuit from **Figure P.2.1** in Multisim:
   a. **Build** the circuit in **Figure P.2.1** in Multisim and find its V_{TH}, I_{SC}, and R_{TH}.
   b. **Build** the equivalent circuits in Multisim.
   c. In Multisim, attach a 1kΩ load resistor (R_L) to the original circuit. **Measure** and **record** the simulated voltage across and current through R_L.
   d. In Multisim, attach 1kΩ load resistors (R_L) to the equivalent circuits. **Measure** and **record** the simulated voltage across and current through R_L.

Electrical Quantity	***V_{TH}***	***R_{TH}***	***I_{SC}***	***Original Circuit***		***Equivalent Circuit***	
				V_{RL}	***I_{RL}***	***V_{RL}***	***I_{RL}***
Calculated							
Simulated							
Percent Error							

Table P.2 – Circuit #2 Data

Part III – Circuit #3 Analysis

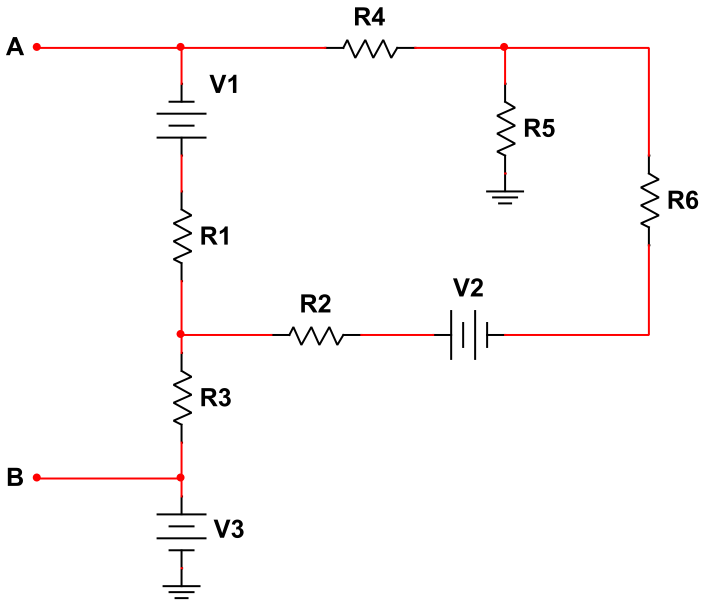

Figure P.3.1 – Circuit #3

In **Figure P.3.1**, the voltage sources **V1 = 3V**, **V2 = 6V**, and **V3 = 9V**.

1. **Calculate** the following from the perspective of terminals A and B. ***Show all work.***
   a. Thévenin voltage (V_{TH})
   b. Thévenin resistance (R_{TH})
   c. Norton current, also called the short-circuit current (I_{SC})
2. **Use** your results to fill in the appropriate values and draw the **Thévenin** and **Norton equivalents** as you did in Part I and Part II.
3. **Attach** a 1kΩ load resistor RL across terminals A and B in **Figure P.3.1** and **calculate** the **voltage across** and **current through R_L** for the new circuit.
4. **Attach** a 1kΩ load resistor RL across terminals A and B in the equivalent circuits that you drew and **calculate** the **voltage across** and **current through R_L** for the new circuits.
7. **Simulate** the circuit from **Figure P.3.1** in Multisim:
   a. **Build** the circuit in **Figure P.3.1** in Multisim and find its V_{TH}, I_{SC}, and R_{TH}.
   b. **Build** the equivalent circuits in Multisim.
   c. In Multisim, attach a 1kΩ load resistor (R_L) to the original circuit. **Measure** and **record** the simulated voltage across and current through R_L.
   d. In Multisim, attach 1kΩ load resistors (R_L) to the equivalent circuits. **Measure** and **record** the simulated voltage across and current through R_L.

Electrical Quantity	***V_{TH}***	***R_{TH}***	***I_{SC}***	***Original Circuit***		***Equivalent Circuit***	
				V_{RL}	***I_{RL}***	***V_{RL}***	***I_{RL}***
Calculated							
Simulated							
Percent Error							

Table P.3 – Circuit #3 Data

LAB

Part I – Circuit #1 Measurements

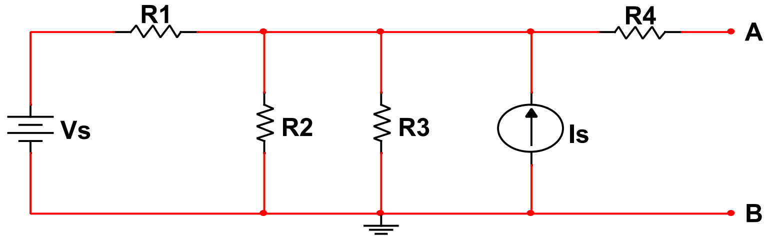

Figure 1.1 – Circuit #1

1. **Build** the circuit from **Part I** of the prelab shown again in **Figure 1.1** on a breadboard.
2. **Measure** the circuit to find **V_{TH}**, **R_{TH}**, and **I_{SC}**.
    a. **V_{TH} Measurement**: **Use** the **DMM** to measure the **voltage** between terminals A and B.
    b. **I_{SC} Measurement**: **Switch** the DMM to current mode and **measure** the **current between** terminals A and B.

    Note: *Remember that the DMM in current mode has a very small internal resistance, such that connecting it directly between terminals A and B short-circuits A to B. This easily gives us the short-circuit current in this case.*

    c. **R_{TH} Measurement**:
        i. **Turn off** and **remove** the DC power supply connections used for Vs and Is. ii. **Use** a wire to connect where the voltage source Vs originally was in your circuit. ***Note:*** *As discussed in the* ***Introduction****, we must "short" the voltage sources. In real life, however, we cannot simply short the power supply as it would overload it.* *****You absolutely must remove the power supply before shorting it.*****
        iii. **Open** the location where Is originally was in your circuit. iv. **Measure** the **resistance** between terminals A and B using the DMM.
3. **Remove** the wire used to short the voltage source and **reconnect** the power supply to the circuit.
4. **Connect** a 1kΩ load resistor RL between terminals A and B and **measure** the **voltage across** and the **current through** RL.
5. **Build** the **Thévenin Equivalent** circuit from **Figure P.1.5** with the load RL attached.
    a. **Set** the power supply to **V_{TH}**.
    b. **Measure** the **voltage across** and the **current through** RL.
6. **Connect** a resistor between terminals A and B that is **equal** to the **Thévenin resistance** (R_{TH}) you calculated. **Measure** the **voltage across** (V_{RTH}) and the **current through** (I_{RTH}) the resistor.
7. **Record** all measured data in the **Table 1.1** and **calculate** the **percent error** between your simulated and measured results.

Electrical Quantity	***V_{TH}***	***R_{TH}***	***I_{SC}***	***V_{RTH}***	***I_{RTH}***	***Original***		***Equivalent***	
						V_{RL}	***I_{RL}***	***V_{RL}***	***I_{RL}***
Calculated									
Simulated									
Measured									
Percent Error									

Table 1.1 – Circuit #1 Data

Part II – Circuit #2 Measurements

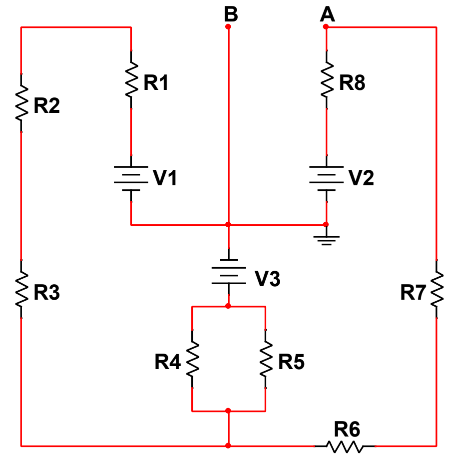

Figure 2.1 – Circuit #2

1. **Build** the circuit from **Part II** of the prelab shown again in **Figure 2.1** on a breadboard.
    a. Refer to the **Introduction** to recall how to set up your circuit and DC power supply with three voltage sources.
2. **Measure** the circuit to find **V_{TH}**, **R_{TH}**, and **I_{SC}**.
    a. **V_{TH} Measurement**: **Use** the **DMM** to measure the **voltage** between terminals A and B.
    b. **I_{SC} Measurement**: **Switch** the DMM to current mode and **measure** the **current between** terminals A and B.
    c. **R_{TH} Measurement**:
        i. **Turn off** and **remove** the DC power supply connections used for Vs and Is.
        ii. **Use** a wire to connect where the voltage source Vs originally was in your circuit.
        iii. **Open** the location where Is originally was in your circuit.
        iv. **Measure** the **resistance** between terminals A and B using the DMM.
3. **Remove** the wire used to short the voltage source and **reconnect** the power supply to the circuit.
4. **Connect** a 1kΩ load resistor RL between terminals A and B and **measure** the **voltage across** and the **current through** RL.
5. **Build** the **Thévenin Equivalent** as you did for Circuit #1.
    a. **Set** the power supply to **V_{TH}**.
    b. **Measure** the **voltage across** and the **current through** RL.
6. **Connect** a resistor between terminals A and B that is **equal** to the **Thévenin resistance** (R_{TH}) you calculated. **Measure** the **voltage across** (V_{RTH}) and the **current through** (I_{RTH}) the resistor.
7. **Record** all measured data in the **Table 2.1** and **calculate** the **percent error** between your simulated and measured results.

Electrical Quantity	***V_{TH}***	***R_{TH}***	***I_{SC}***	***V_{RTH}***	***I_{RTH}***	***Original***		***Equivalent***	
						V_{RL}	***I_{RL}***	***V_{RL}***	***I_{RL}***
Calculated									
Simulated									
Measured									
Percent Error									

Table 2.1 – Circuit #2 Data

Part III – Thévenin Design Problem

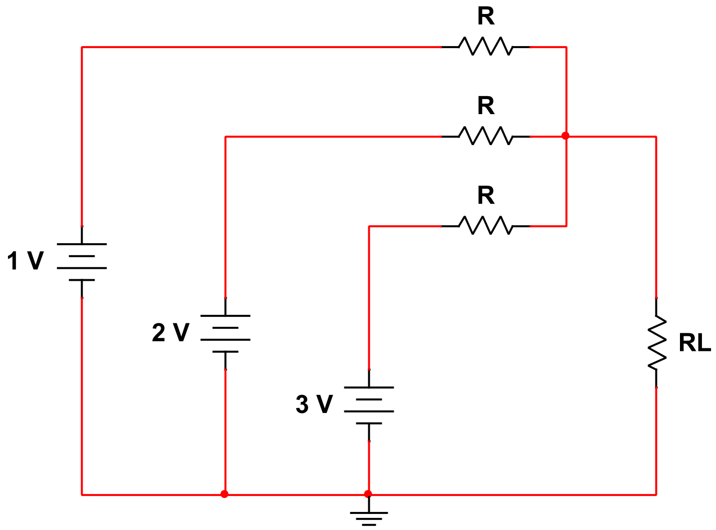

Figure 3.1 – Design Problem

1. **Determine** the resistances R and RL such that the following specifications are met:
   - **Voltage across** RL is equal to ½V_{TH}
   - **Maximum power** dissipated by RL is 3mW

   Hint: *Start by removing the load resistor and finding the Thévenin equivalent circuit as you have done for the previous circuits in this lab. You will need to find the value for RL before solving for R. Also, note that all three resistors labeled R are the same value resistance.*
2. For your circuit with the calculated value of R, is there any other resistor that could be substituted in for RL to increase the power dissipated by the load? **Explain**.

POST-LAB ANALYSIS

Include answers to the following questions in the Analysis and Discussion section of your lab report.

1. When you attach the 1kΩ load resistor (RL) to any of the circuits in the lab, is the voltage across RL equal to V_{TH}? Is the current through RL equal to I_{SC}?
2. Is there any load resistor (RL) that you could attach to the circuit to achieve both V_{TH} and I_{SC} at the same time? Explain.
3. When you attach the Thévenin equivalent resistance (R_{TH}) to any of the circuits in the lab, is there a relationship between V_{TH} and V_{RTH}? What about I_{SC} and I_{RTH}?

Include the following in the Conclusion of your lab report:

1. Discuss how Thévenin's Theorem helps you simplify the circuit analysis. For example, consider this idea when writing your conclusion: is it easier to use the Thevenin equivalent circuit to determine the voltage across and current through the load resistor (RL) or is it easier to use the original circuit?
2. What is the purpose of the Thévenin Equivalent circuit?