THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

SCHOOL OF ENGINEERING AND APPLIED SCIENCE
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
ECE 2110: CIRCUIT THEORY LABORATORY

Experiment #10:
Passive Filter Design

EQUIPMENT

Lab Equipment	*Equipment Description*
(1) Function Generator	Supplied by the AD2
(1) Digital Multimeter (DMM)	Harbor Freight Model 63759 Handheld Digital Multimeter
(1) Digital Oscilloscope	Supplied by the AD2
(1) Breadboard	Prototype Breadboard
(1) Test Leads	Banana to Alligator Lead Set

Table 1 – Equipment List

COMPONENTS

Type	*Value*	*Symbol Name*	*Multisim Part*	*Description*
Resistor	3.3kΩ	R	Basic/Resistor	---
Resistor	510Ω	R_2	Basic/Resistor	---
Capacitor	820pF	C	Basic/Capacitor	Ceramic Disk, 821J
Inductor	4.7mH	L	Basic/Inductor	---

Table 2 – Component List

OBJECTIVES

- Find the frequency response of a series RC and RL circuit
- Plot the magnitude and phase response of a series RC and RL circuit
- Design, build, and test a low-pass filter
- Design, build, and test a high-pass filter
- Find the frequency response of a series and parallel resonance circuit
- Plot the magnitude and phase response of a series and parallel resonance circuit
- Design, build, and test a band-pass filter

INTRODUCTION

This lab will focus on understanding the behavior of common filters and how we can create filters with simple passive components such as capacitors, inductors, and resistors.

Filters

An electric filter modifies the frequency content of a signal. **Figure 1** shows the four main types of filters: **high-pass** (HPF), **low-pass** (LPF), **band-pass** (BPF), and **band-stop** (notch). A low-pass filter allows low frequencies to pass to the load while attenuating high frequencies. A high-pass filter allows high frequencies to pass while attenuating low frequencies. A band-pass filter allows a range of frequencies to pass while attenuating frequencies outside of that range. A band-stop filter attenuates a range of frequencies while passing frequencies outside of that range.

In **Figure 1**, the x-axes represent frequency (ω) in radians per second. By convention, frequency is represented by the variable ω when its units are radians per second and f when its units are Hertz. The y-axes represent the gain of each filter. In this instance, gain is defined as the voltage across the load divided by the input voltage. As is shown in the figure, the gain of a filter is different at different frequencies.

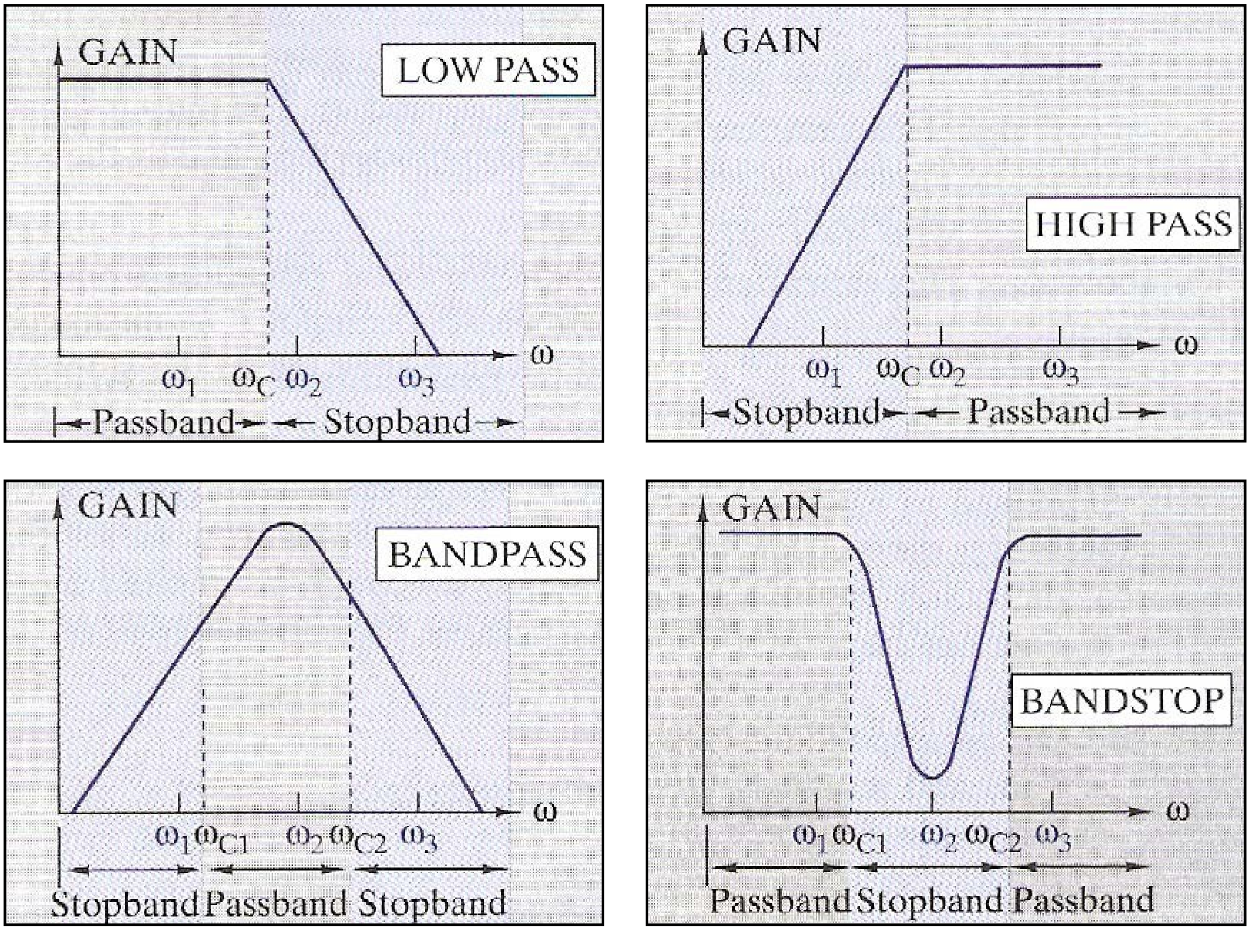

Figure 1 – Gain Responses (Thomas et al., page 602)

Common Filter Terms

The range of frequencies that are attenuated is called the **stopband**. The range of frequencies that pass to the load is called the **passband**. The **cutoff frequency** (ω_C or f_C) is the frequency at the transition between the stopband and passband (band-pass and band-stop filters will have two cutoff frequencies). An **ideal filter** passes frequencies in the passband without modifying their magnitude (Gain = 1) and completely attenuates frequencies in the stopband (Gain = 0). However, **ideal filters do not exist in practice**. One convention is to define cutoff frequencies as the frequency at which the magnitude of the voltage at the load is decreased by **3dB** from its maximum value $\left(\frac{V_{max}}{\sqrt{2}}\right)$, called the **-3dB frequency**.

There are other ways to define the cutoff frequency, so when reading or specifying ω_C, make sure that you understand which definition is being used. With respect to a filter, a **decibel** (dB) is defined as ten times the logarithm to base 10 of the ratio of the output power to the input power. When the input and output powers are delivered to an equal resistance, a decibel can be defined with respect to the voltage gain of the filter. This derivation is shown in **Equation 1**. Using this definition, it can be shown that a 3dB reduction in voltage is approximately equal to a reduction of $\frac{1}{\sqrt{2}}$ in voltage or a reduction of **half the power.**

$$\#\,of\ dB = 10\log_{10}\frac{P_{out}}{\underset{P}{P_{in}}} = 10\log_{10}\frac{\frac{V_{Out}^2}{R}}{\frac{V_{in}^2}{R}} = 10\log_{10}\frac{V_{out}}{V_{in}}\frac{V_{out}}{V_{in}} = 10\log_{10}\frac{V_{out}}{V_{in}} + 10\log_{10}\frac{V_{out}}{V_{in}} = \mathbf{20\ \log_{10}}\frac{\boldsymbol{V_{out}}}{\boldsymbol{V_{in}}}$$

Equation 1 – Decibel Derivation (Thomas et al., page 603)

The **center frequency** (ω_0 or f_0) is the frequency where the voltage at the load is at its **maximum** value. The **bandwidth** (B) of a filter is the difference between the two cutoff frequencies. The **quality factor** (Q) is the ratio of the center frequency to the bandwidth $\left(Q = \frac{\omega_0}{B}\right)$. The **gain function** of a filter is the ratio of the magnitude of the frequency response of the filter at the load to the magnitude of the frequency response of the source.

***Note:** For passive circuits, the gain must be **less than one**.*

As an example, the magnitude and phase of the voltage at the load of a series RL circuit, given a source voltage of 1∠0°V, is shown in Figure P1. The red line illustrates the voltage across the inductor, and the blue line is the voltage across the resistor (load). The magnitude plot shows that the circuit has a cutoff frequency (-3dB frequency) of approximately 30kHz, a passband from 0 to 30kHz, and a stopband from 30kHz to Infinity.

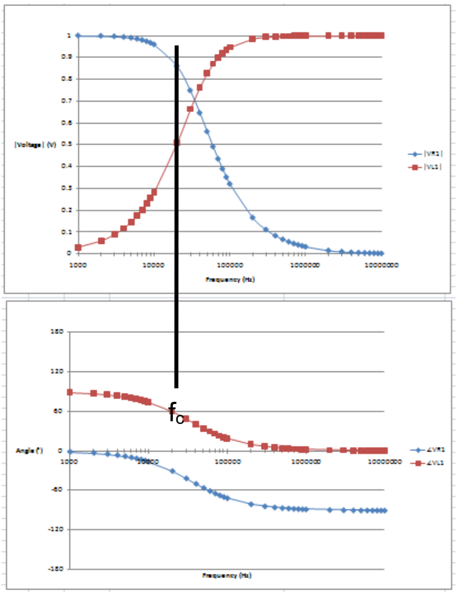

Figure P1: Low-Pass Filter - magnitude (top), phase(bottom)

PRELAB

Review the provided Excel spreadsheet (lab10_example.xlsx). It demonstrates a partial solution to Part I of the prelab. Using this spreadsheet as an example, produce similar spreadsheets for Parts II-IV.

Part I – Series RC Circuit

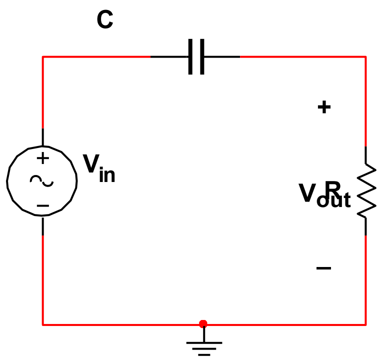

Figure P.1 – Series RC Circuit

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_C}$ and $\mathbf{V_R}$ associated with C and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_C}$, $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_C + V_R = V_{in}$ for all frequencies using Excel.

   Hint: *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **C = 820pF**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

Part II – Series RL Circuit

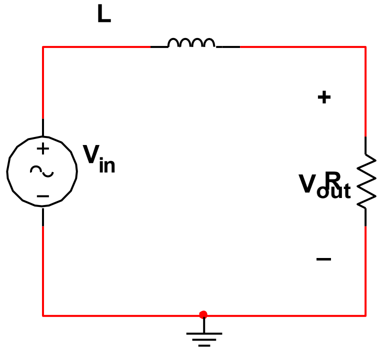

Figure P.2 – Series RL Circuit

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_L}$ and $\mathbf{V_R}$ associated with L and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_L}$, $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_L + V_R = V_{in}$ for all frequencies using Excel.

   Hint: *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

Part III – Series Resonant Circuit

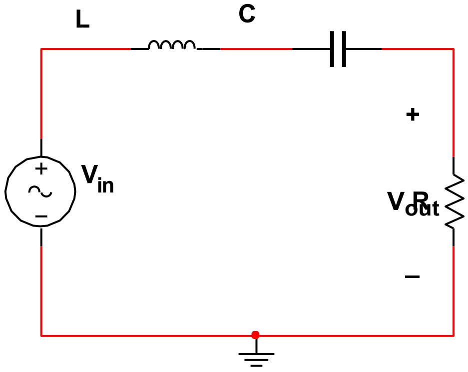

Figure P.3 – Series Resonant Circuit

1. **Compute** the equivalent impedance $\mathbf{Z_{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$ associated with L, C, and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V_L}$, $\mathbf{V_C}$, and $\mathbf{V_R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that $V_L + V_C + V_R = V_{in}$ for all frequencies using Excel.

   Hint: *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **C = 820pF**
   - **R = 3.3kΩ**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V_{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

Part IV – Parallel Resonant Circuit

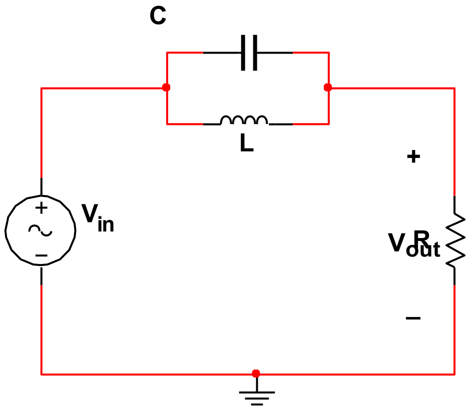

Figure P.4 – Parallel Resonant Circuit

1. **Compute** the equivalent impedance $\mathbf{Z}_{\mathbf{TH}}$ for the circuit in **Figure P.1**.
2. Establish the **general equations** for the phasor voltages $\mathbf{V}_\mathbf{L}$, $\mathbf{V}_\mathbf{C}$, and $\mathbf{V}_\mathbf{R}$ associated with L, C, and R (leave in rectangular form).
3. **Use Excel** to calculate the **amplitudes** and **phase differences** of the phasor voltages ($\mathbf{V}_\mathbf{L}$, $\mathbf{V}_\mathbf{C}$, and $\mathbf{V}_\mathbf{R}$) for frequencies from 1kHz to 10MHz. Use the following increments as shown in the sample Excel sheet: 1kHz, 2kHz,…, 9kHz,10kHz, 20kHz,…, 90kHz, 100kHz, 200kHz,..., 900kHz,1MHz, 2MHz,…,10MHz. Verify that (V_L or V_C) + V_R = V_{in} for all frequencies using Excel.

   Hint: *Review the Excel help files for the commands COMPLEX, IMDIV, IMPRODUCT, IMREAL, IMAGINARY, IMABS, IMTAN2, and PI.*
   - $\mathbf{V_{in} = 1 \angle 0°V}$
   - **L = 4.7mH**
   - **C = 820pF**
   - **R = 510Ω**
4. **Plot** a **graph** of **amplitudes versus frequency** in **Excel** (use a legend to identify the different curves). **Find** the **-3dB frequency** for the R curve ($\mathbf{V}_{\mathbf{out}}$).
5. **Plot** a **graph** of **phase differences versus frequency** in **Excel** (include a legend).

LAB

Part I – Series RC Circuit Measurement

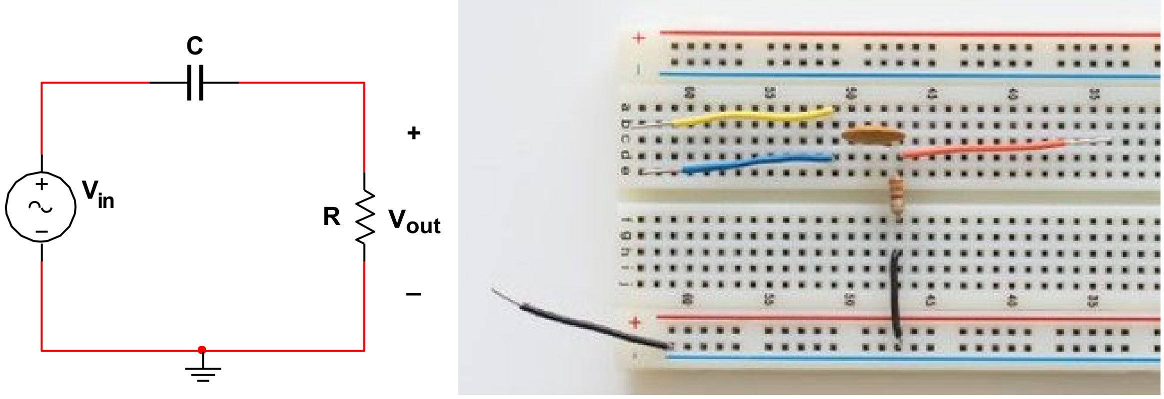

Figure 1.1 – Series RC Circuit

1. **Build** the circuit in **Figure 1.1** on a breadboard using the following components:
    - **V_{in} = 1V_{pk}**
    - **C = 820pF**
    - **R = 3.3kΩ**
2. **Open WaveForms.** You will need the Wavegen tab and the Oscilloscope tab.
3. **Measure V_{out}** (magnitude and phase) for different frequencies from 1kHz to 10MHz.
    a. To do this, make sure the amplitude of the signal generated by the Wavegen is at 1V.
    b. Next, connect Channel 1 of the Oscilloscope (Orange Wire/Pin 1+) to the node where Vin and the capacitor connect. Then connect Channel 2 of the Oscilloscope (Blue Wire/ Pin 2+) to the node where the capacitor and resistor connect.
    c. Set up the Oscilloscope. Using the proper "Measurements" tab, add the Magnitude of Channel 2, and add a global measurement to measure the phase angle between the two signals.
    d. For each frequency value, you will have to change the frequency value within the Wavegen tab.
4. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
5. **Plot** the **phase difference versus frequency** in Excel using your collected data.

Frequency	***Magnitude***	***Phase***
1kHz		
10kHz		
20kHz		
40kHz		
60kHz		
80kHz		
100kHz		
200kHz		
300kHz		
500kHz		
600kHz		
800kHz		
1MHz		
5MHz		
10MHz		

Part II – Series RL Circuit Measurement

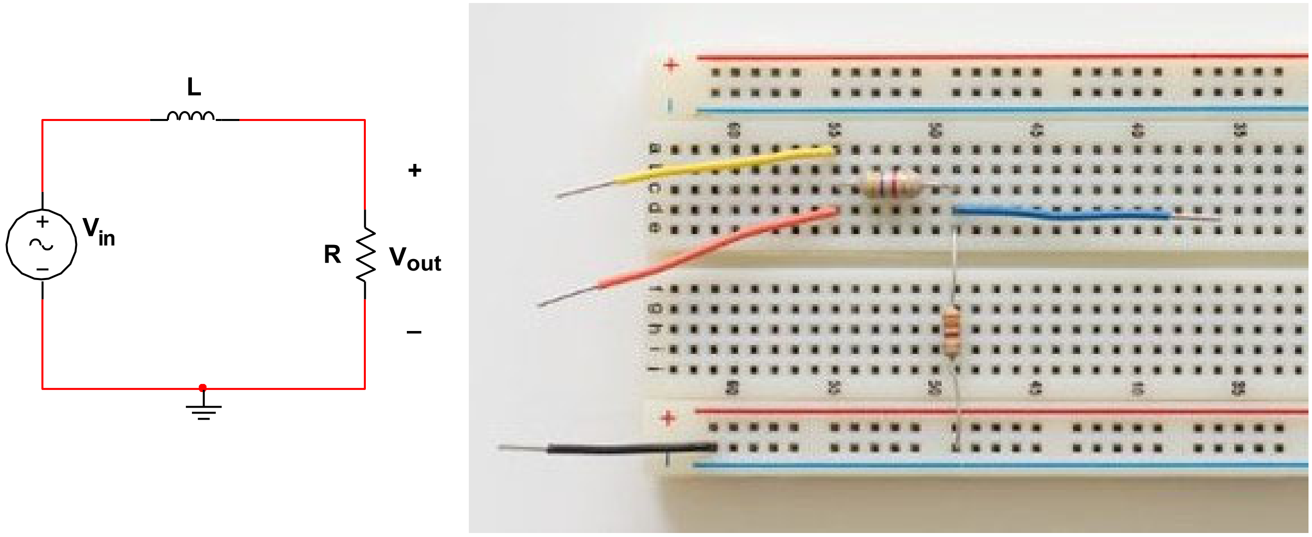

Figure 2.1 – Series RL Circuit

1. **Build** the circuit in **Figure 2.1** on a breadboard using the following components:
    - **V_{in} = $1V_{pk}$**
    - **L = 4.7mH**
    - **R = 3.3kΩ**
2. **Measure V_{out}** (magnitude and phase) for different frequencies from 1kHz to 10MHz. Use the same setup and procedure as Part I.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.

Frequency	***Magnitude***	***Phase***
1kHz		
10kHz		
20kHz		
40kHz		
60kHz		
80kHz		
100kHz		
200kHz		
300kHz		
500kHz		
600kHz		
800kHz		
1MHz		
5MHz		
10MHz		

Table 2.1 – Series RL Circuit Data

Part III – Series Resonant Circuit Measurement

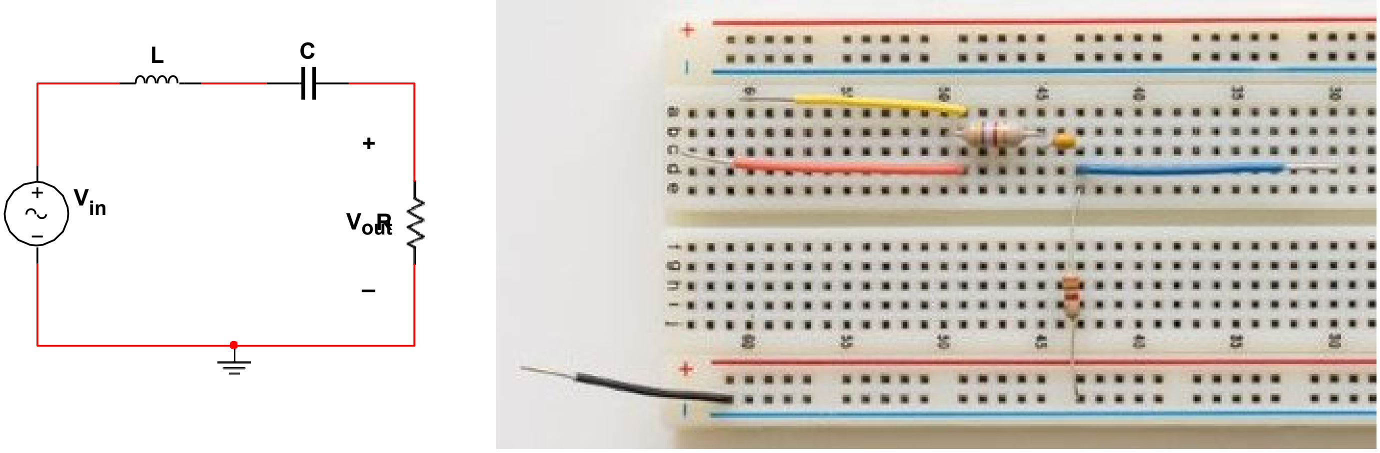

Figure 3.1 – Series Resonant Circuit

1. **Build** the circuit in **Figure 3.1** on a breadboard using the following components:
    - **V_{in} = 1V_{pk}**
    - **L = 4.7mH**
    - **C = 820pF**
    - **R = 3.3kΩ**
2. **Measure V_{out}** (magnitude and phase) for different frequencies from 1kHz to 10MHz. Use the same setup and procedure as Part **I**.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.
5. **Find** the cutoff frequencies (**ω_{C1} and ω_{C2}**), Bandwidth (**B**), center frequency (**ω_0**) and Quality factor (**Q**).

Frequency	*Magnitude*	*Phase*
1kHz		
5kHz		
10kHz		
20kHz		
40kHz		
70kHz		
80kHz		
90kHz		
100kHz		
200kHz		
400kHz		
500kHz		
1MHz		
5MHz		
10MHz		

Table 3.1 – Series Resonant Circuit Data

Part IV – Series Parallel Circuit Measurement

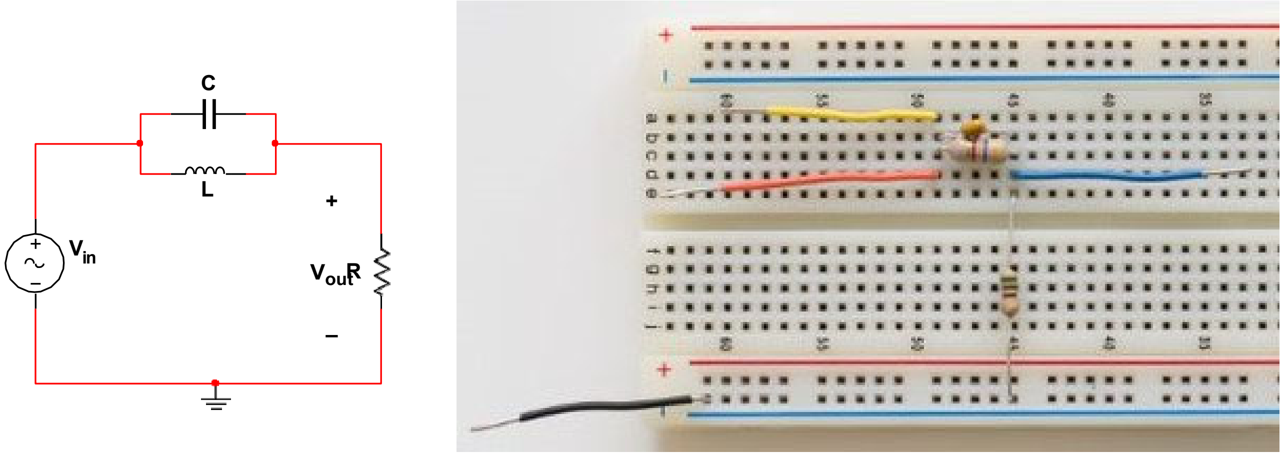

Figure 4.1 – Parallel Resonant Circuit

1. **Build** the circuit in **Figure 4.1** on a breadboard using the following components:
    - **V_{in} = 1V_{pk}**
    - **L = 4.7mH**
    - **C = 820pF**
    - **R = 510Ω**
2. **Measure V_{out}** (magnitude and phase) for different frequencies from 1kHz to 10MHz. Use the same setup and procedure as Part I.
3. **Plot magnitude versus frequency** in Excel using your collected data. **Find** the -3dB frequency.
4. **Plot** the **phase difference versus frequency** in Excel using your collected data.
5. **Find** the cutoff frequencies (**ω_{C1} and ω_{C2}**), Bandwidth (**B**), center frequency (**ω_0**) and Quality factor (**Q**).

Frequency	***Magnitude***	***Phase***
1kHz		
5kHz		
10kHz		
30kHz		
40kHz		
60kHz		
80kHz		
90kHz		
100kHz		
200kHz		
100kHz		
500kHz		
1MHz		
5MHz		
10MHz		

Table 4.1 – Series Parallel Circuit Data

Part V – Low-Pass Filter Design

1. Use Multisim to **design** and **simulate** a **high-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - -3dB Frequency: 500Hz
    - Tolerances: 5%
2. **Build and test this circuit.**

Part VI – High-Pass Filter Design

1. Use Multisim to **design** and **simulate** a **high-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - -3dB Frequency: 20kHz
    - Tolerances: 5%
2. **Build and test this circuit.**

Part VII – Band-Pass Filter Design

1. Use Multisim to **design** and **simulate** a **band-pass filter** that meets the following specifications: ***Show all steps of your design.***
    - Applied Voltage: $1V_{rms}$
    - Quality Factor: 1
    - Bandwidth: 15kHz
    - Tolerances: 5%
2. **Build and test this circuit.**

POST-LAB ANALYSIS

1. **Compare** the calculated results to the measured results and explain any and all differences.
2. **Describe** the motivation behind defining the cutoff frequency at the point where the gain is -3dB as opposed to -4dB or -5dB.
3. **What type of filter** would you want to implement if you observed **high frequency noise** in your voltage signal?
4. **Describe** a situation where a band-pass filter would be desired.
5. **Does** it make sense to define the bandwidth of a high-pass filter? **Explain**.
6. **Describe** the relationship between ω_C and f_C. Be sure to include the **mathematical relationship**.
7. **Hum noise** is a common phenomenon in electronic devices especially hi-fi equipment. The noise comes from the line (110 Vacrms @ 60 Hz). Using the information you have learned so far, how could you eliminate this noise?
8. **Describe** how the quality factor (Q) is used to distinguish between narrow-band and wide-band filters.

REFERENCES

[1] Thomas, Roland E., Albert J. Rosa, and Gregory J. Toussaint. *The Analysis and Design of Linear Circuits*. 7th ed. Hoboken, NJ: Wiley, 2012.