# THE GEORGE WASHINGTON UNIVERSITY 

WASHINGTON, DC

## School of Engineering and Applied Science Department of Electrical and ComputerEngineering ECE 2110: Circuit Theory Laboratory

Experiment \#4:<br>Voltage Division, Circuit Reduction, Ladders, and Bridges

## EqUIPMENT

| Lab Equipment | Equipment Description |
| :--- | :---: |
| (1) DC Power Supply | Supplied by the AD2 |
| (1) Digital Multimeter (DMM) | Harbor Freight Model 63759 Handheld Digital Multimeter |
| (1) Breadboard | Prototype Breadboard |
| (1) Test Leads | Banana to Alligator Lead Set |

Table 1 - Equipment List

## Components

| Type | Value | Symbol Name | Multisim Part | Description |
| :---: | :---: | :---: | :---: | :---: |
| Resistor | $470 \Omega$ | $\mathrm{R}_{1}$ | Basic/Resistor | --- |
| Resistor | $560 \Omega$ | $\mathrm{R}_{2}$ | Basic/Resistor | --- |
| Resistor | $680 \Omega$ | $\mathrm{R}_{3}$ | Basic/Resistor | --- |
| Resistor | $750 \Omega$ | $\mathrm{R}_{4}$ | Basic/Resistor | --- |
| Resistor | $820 \Omega$ | $\mathrm{R}_{5}$ | Basic/Resistor | --- |
| Resistor | $910 \Omega$ | $\mathrm{R}_{6}$ | Basic/Resistor | --- |
| Potentiometer | $10 \mathrm{k} \Omega$ | $\mathrm{R}_{7}$ | Basic/Resistor | --- |
| LED Bar | Green | LEDs | LED/Bar_LED_Green_Ten | Bar of 10 Green LEDs |

Table 2 - Component List

## Objectives

- To calculate and measure the total resistance in a series/parallel circuit
- To calculate and measure the total current in a series/parallel circuit for a given applied voltage
- To calculate the expected total power dissipated in a series/parallel circuit from nominal and measured values
- To calculate and measure the voltage drop across and the current through all components of a series/parallel circuit
- To calculate the power dissipated by each component of a series/parallel circuit from measured data
- Design, build and test a voltage ladder
- Use a Wheatstone Bridge circuit as a detector of unknown resistances


## INTRODUCTION

## Voltage Division

The voltage divider circuit is one of the most important and fundamental circuits that you will encounter in Circuit Theory. There are countless applications for voltage dividers, and you will find that throughout the semester, many calculations become simpler if you see the circuit as a basic voltage divider. A voltage divider is a linear circuit whose output voltage is some fraction of its input voltage. In other words, a voltage divider allows us to turn a large voltage into a smaller one. Voltage division refers to the splitting of a voltage among the components of the voltage divider.


Figure 1 - Voltage Divider

$$
V_{\text {out }}=\frac{R_{2}}{R_{1}+R_{2}} \cdot V_{\text {in }}
$$

Equation 1 - Voltage Divider Equation

## Potentiometer

A potentiometer is a passive device that acts as a variable resistor or voltage divider. Potentiometers have three terminals and some type of rotary component that can be turned clockwise and counterclockwise to adjust the resistance. The resistance between the outer two terminals is constant and always equal to the nominal value of the potentiometer. The middle terminal is connected to the wiper shown in Figure 4 that can be adjusted to change the resistance relationship between the two halves of the device. Figure 2 shows the schematic symbol for a potentiometer, including the third middle terminal. Think of the potentiometer in Figure 4 as two resistors connected in series with each other, $R_{1}$ between terminals $A$ and W, $\mathrm{R}_{2}$ between W and B , similar to the voltage divider of Figure 1. When the center dial is turned counterclockwise until it stops, $R_{1}$ would be $0 \Omega$ and $R_{2}$ would equal to the nominal value of the potentiometer. As the dial is turned clockwise, $R_{1}$ increases and $R_{2}$ decreases.


Figure 2 - Potentiometer Symbol


Figure 3 - Potentiometer


Figure 4 - Potentiometer Diagram

## Voltage Ladder

A voltage ladder is a simple electrical circuit comprised of multiple resistors in series used to create multiple voltage points between 0 V and the source voltage. Similar to a voltage divider, the source voltage is split across the resistors as determined by the ratio of the individual resistor to the entire equivalent resistance.


Figure 5 - Voltage Ladder

## Wheatstone Bridge

A Wheatstone Bridge is an electrical bridge circuit used to determine and unknown resistance. The Wheatstone Bridge circuit consists of four resistances, three of which must be known and one that is unknown. By balancing the two legs of the bridge circuit and deriving the resistive relationship, the unknown resistance can be quickly determined. In Figure 6, $\mathrm{R}_{\mathrm{x}}$ is the unknown resistance we are trying to find, $\mathrm{R}_{3}$ is a variable resistor such as a potentiometer that we can adjust, and $R_{1}$ and $R_{2}$ are known resistances.


Figure 6 - Wheatstone Bridge

## Prelab

## Part I - Simplifying a Parallel Circuit



Figure P. 1 - Parallel Circuit

1. Simplify the circuit in Figure $\mathbf{P} .1$ to find the voltage across, current through, and power dissipated by each resistor.
a. Find the equivalent resistance $\mathbf{R}_{\mathrm{EQ}}$ from the perspective of the voltage source $\mathrm{V}_{\mathrm{S}}$ in Figure P.1. Leave your answer in terms of $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}$, and $\mathrm{R}_{4}$. Show all work.
b. Calculate the actual equivalent resistance $\mathbf{R E Q}_{\text {EQ }}$ using the resistor values given in Table 2.
c. Calculate the total current drawn by $\mathrm{ReQ}_{\mathrm{EQ}}$ and the total power dissipated by $\mathrm{R}_{\mathrm{EQ}}$ assuming $\mathrm{V}_{\mathrm{s}}=5 \mathrm{~V}$.
d. Calculate the voltage drop across each resistor, the current through each resistor, and the power dissipated by each resistor.
e. Record all your results in TableP.1.
2. Simulate the circuit in Figure P. 1 in Multisim.
a. Find the voltage drop across each resistor, the current through each resistor, and the power dissipated by each resistor.
b. Record all values in Table P.1.
3. Calculate the percent error between your calculated and simulated results.
a. Record the error in Table P.1.

| Electrical Quantity | Resistor |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R_{E Q}$ | $R_{1}$ | $R_{\mathbf{2}}$ | $\boldsymbol{R}_{3}$ | $\boldsymbol{R}_{\mathbf{4}}$ |  |
| Voltage <br> (V) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |
| Current <br> (mA) | Calculated |  | Simulated |  |  |  |
|  | Cercent Error |  |  |  |  |  |
|  | Calculated |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |

Table P. 1 - Prelab Data Table 1

## Part II - Simplifying a Series Circuit



Figure P. 2 - Series Circuit

1. Simplify the circuit in Figure P. 2 in order to find the voltage across, current through, and power dissipated by each resistor.
a. Find the equivalent resistance $\mathbf{R}_{\mathrm{EQ}}$ from the perspective of the voltage source $\mathrm{V}_{\mathrm{S}}$ in Figure P.2. Leave your answer in terms of $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}$, and $\mathrm{R}_{4}$. Show all work.
b. Calculate the actual equivalent resistance $\mathbf{R}_{\mathrm{EQ}}$ using the resistor values given in Table 2.
c. Calculate the total current drawn by $\mathrm{R}_{\mathrm{EQ}}$ and the total power dissipated by $\mathrm{R}_{\mathrm{EQ}}$ assuming Vs=5V.
d. Calculate the voltage drop across each resistor, the current through each resistor, and the power dissipated by each resistor.
e. Record all your results in Table P.2.
2. Simulate the circuit in Figure P. 2 in Multisim.
a. Find the voltage drop across each resistor, the current through each resistor, and the power dissipated by each resistor.
b. Record all values in Table P. 2.
3. Calculate the percent error between your calculated and simulated results.
a. Record the error in Table P.2.

| Electrical Quantity | Resistor |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R_{E Q}$ | R1 | R2 | R3 | R4 |  |
| Voltage <br> (V) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |
| Current <br> (mA) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
| Power <br> (mW) | Calculated | Simulated |  |  |  |  |
|  | Percent Error |  |  |  |  |  |

Table P. 2 - Prelab Data Table 2

## Part III - Simplifying a Series-Parallel Circuit



Figure P. 3 - Series-Parallel Circuit

1. Simplify the circuit in Figure $\mathbf{P} .3$ in order to find the voltage across, current through, and power dissipated by each resistor
a. Find the equivalent resistance $\mathbf{R}_{\mathrm{EQ}}$ from the perspective of the voltage source $\mathrm{V}_{\mathrm{S}}$ in Figure P.3. Leave your answer in terms of $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}, \mathrm{R}_{4}$. $\mathrm{R}_{5}$ and $\mathrm{R}_{6}$. Show all work.
b. Calculate the actual equivalent resistance $\mathrm{R}_{\mathrm{EQ}}$ using the resistor values given in Table 2.
c. Calculate the total current drawn by $\mathbf{R E Q}_{\mathrm{EQ}}$ and the total power dissipated by $\mathrm{R}_{\mathrm{EQ}}$ assuming $\mathrm{Vs}=5 \mathrm{~V}$.
d. Calculate the voltage drop across each resistor, the current through each resistor, and the power dissipated by each resistor.
e. Record your results in TableP.3.
2. Simulate the circuit in Figure P. 3 in Multisim
a. Find the voltage drop across each resistor, the current through each resistor, andthe power dissipated by each resistor.
b. Record all values in Table P.3.
3. Calculate the percent error between your calculated and simulated results.
a. Record the error in Table P.3.

| Electrical Quantity | Resistor |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R_{E Q}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ | $R_{5}$ | $\boldsymbol{R}_{6}$ |  |
| Voltage <br> (V) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |  |  |
| Current <br> (mA) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |  |  |
| Power <br> (mW) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Percent Error |  |  |  |  |  |  |  |

Table P. 3 - Prelab Data Table 3

## Part IV - Wheatstone Bridge

The circuit below in Figure P. 4 is a Wheatstone Bridge. As discussed in the Introduction to this lab, the purpose of the Wheatstone Bridge is to use three known resistors $\left(R_{1}, R_{2}\right.$, and $\left.R_{3}\right)$ to find an unknown resistance $R_{x}$. We need to generate an algebraic expression for resistor $R x$ in terms of the known resistors. Use the following steps to solve the bridge circuit for $\mathrm{R}_{\mathrm{x}}$ :


Figure P. 4 - Wheatstone Bridge Circuit

1. Solve for $V_{A}$ in terms of $R_{1}, R_{2}$, and $V_{S}$. (Hint: $V_{A}$ is just the voltage across $R_{2}$ )
2. Solve for $V_{B}$ in terms of $R_{3}, R x$, and $V_{S}$. (Hint: $V_{B}$ is just the voltage across $R_{X}$ ) Note: The bridge is said to be "balanced" when $V_{A}=V_{B}$.
3. Solve for $R_{x}$ by setting your equations for $V_{A}$ and $V_{B}$ equal to each other.

Note: $V_{S}$ should drop out of the equation if you have done everything correctly.

## LAB

Part I - Parallel Circuit Measurements



Figure 1.1 - Parallel Circuit


Figure 1.2 - Measuring Total Current

1. Build the circuit in Figure $\mathbf{1 . 1}$ on a breadboard. Before connecting the AD2 to the circuit, use the DMM to measure the equivalent resistance $\mathbf{R}_{E Q}$ of the circuit.
2. Connect the AD2 $\left(\mathbf{V}_{\mathbf{s}}=\mathbf{5 V}\right)$ to the circuit, turn on WaveForms, and click "Supplies".
3. Click the play button in WaveForms to turn on the voltage source.
4. Measure the voltage across each resistor and the current through each resistor with the DMM. Note: Remember that it is impossible to measure the current across a resistor. You must use the DMM differently when measuring current. You can check back to Lab 3 if you need a reminder.
5. Measure the total current supplied by the AD2. MAKE SURE THE AD2 IS NOT SUPPLYING A VOLTAGE TO YOUR CIRCUIT FIRST. To do this, disconnect the positive and ground wires from the resistors. Then take those wires and put them anywhere else on the breadboard, just not on the same rail. Set your DMM to measure current, connect the red lead to the positive terminal, the black lead to ground, and then turn on the power supply. Figure 1.2 shows the result of this process.
6. Calculate the power dissipated by each resistor using the measured voltage and current.
7. Record all data in the Measured sections of Table 1.1.
8. Compute the percent error between your calculated and measured results and record it below.

| Electrical Quantity | Resistor |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R_{E Q}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ |  |
| Voltage <br> (V) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Measured |  |  |  |  |  |
| Current <br> (mA) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Measured |  |  |  |  |  |
| Power <br> (mW) | Calculated |  | Simulated |  |  |  |
|  | Measured |  |  |  |  |  |
|  | Voltage | Current |  |  |  |  |

Table 1.1 - Parallel Circuit Data

## Part II - Series Circuit Measurements



Figure 2.1 - Series Circuit

1. Build the circuit in Figure 2.1. Before connecting the AD2 to the circuit, use the DMM to measure the equivalent resistance $\mathrm{R}_{\mathrm{EQ}}$ of the circuit.
2. Connect the AD2 $\left(\mathbf{V}_{\mathbf{s}}=\mathbf{5 V}\right)$ to the circuit. Ensure WaveForms is still open on your computer.
3. Click the play button in WaveForms to turn on the voltage source.
4. Measure the voltage across each resistor and the current through each resistor with the DMM.
5. Measure the total current supplied by the AD2.
6. Calculate the power dissipated by each resistor using the measured voltage and current.
7. Record all data in the Measured sections of Table 2.1.
8. Compute the percent error between your calculated and measured results and record it below.

| Electrical Quantity |  | Resistor |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $R_{E Q}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ |
| Voltage (V) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Measured |  |  |  |  |  |
| Current (mA) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Measured |  |  |  |  |  |
| Power (mW) | Calculated |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |
|  | Measured |  |  |  |  |  |
| Percent Error (\%) | Voltage |  |  |  |  |  |
|  | Current |  |  |  |  |  |
|  | Power |  |  |  |  |  |

Table 2.1 - Series Circuit Data

## Part III - Series-Parallel Circuit Measurements



Figure 3.1 - Series-Parallel Circuit

1. Build the circuit in Figure 3.1. Before connecting the power supply to the circuit, use the DMM to measure the equivalent resistance $\mathbf{R}_{\mathrm{EQ}}$ of the circuit.
2. Connect the power supply $\left(\mathbf{V}_{\mathbf{s}}=\mathbf{5 V}\right)$ to the circuit.
3. Measure the voltage across each resistor and the current through each resistor with the DMM.
4. Measure the total current supplied by the powersupply.
5. Calculate the power dissipated by each resistor using the measured voltage andcurrent.
6. Record all data in the Measured sections of Table 3.1.
7. Compute the percent error between your calculated and measured results and record it below.

| Electrical Quantity |  | Resistor |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $R_{E Q}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ | $R_{5}$ | $\boldsymbol{R}_{6}$ |
| Voltage (V) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Measured |  |  |  |  |  |  |  |
| Current (mA) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Measured |  |  |  |  |  |  |  |
| Power (mW) | Calculated |  |  |  |  |  |  |  |
|  | Simulated |  |  |  |  |  |  |  |
|  | Measured |  |  |  |  |  |  |  |
| Percent Error (\%) | Voltage |  |  |  |  |  |  |  |
|  | Current |  |  |  |  |  |  |  |
|  | Power |  |  |  |  |  |  |  |

Table 3.1 - Series-Parallel Circuit Data

## Part IV - Design, Build, and Test a Voltage Ladder

In this part of the lab, you are asked to design a voltage ladder using the concept of Voltage Division.

## Design Specifications:

- $\mathbf{P}_{\text {total }}$ : $\leq 86 \mathrm{~mW}$
- $\mathrm{V}_{\mathrm{s}}$ : $5 \mathrm{~V} \pm 5 \%$
- $\mathrm{V}_{\mathrm{AB}}: 1.724 \mathrm{~V} \pm 5 \%$
- $\mathbf{V}_{\mathrm{BC}}$ : $0.345 \mathrm{~V} \pm 5 \%$
- $\mathbf{V}_{\mathrm{CD}}: 1.724 \mathrm{~V} \pm 5 \%$
- $\mathrm{V}_{\mathrm{DE}}$ : $1.207 \mathrm{~V} \pm 5 \%$


Figure 4.1 - Voltage Ladder

1. Using the circuit in Figure 4.1, find the appropriate values for $R_{1}, R_{2}, R_{3}$, and $R_{4}$ to build a voltage ladder that meets the Design Specifications. (Hint: A good first step would be to examine the relationship between Vs, $P_{\text {total, }}$, and total current.)
Note: $R_{1}, R_{2}, R_{3}$, and $R_{4}$ are for you to calculate. Do not use the values from Table 2 that were used in earlier parts of this lab.
2. After designing the voltage ladder, build it on a breadboard.
3. Turn on the Power Supply in WaveForms.
4. Measure the voltage drop across each resistor and compare it to the value in the specifications.
a. Record your results in Table 4.1.
5. Connect the circuit to your green LED bar from points $B, C$, and $D$. Wire from nodes $B, C$, and $D$ to the positive side (anode) of three individual LEDs in the bar. The positive side of the bar has the bar's serial code on it.
6. Connect the negative side (cathode) of the LEDs directly to ground. Vary the supply voltage from 0 V to 5 V (feel free to experiment).
Question: What happens to the intensity of the LEDs as you vary the supply voltage? Do you notice a pattern?

| Quantity | Specified | Measured | Percent Error (\%) |
| :---: | :--- | :--- | :--- |
| $\boldsymbol{P}_{\text {total }}$ |  |  |  |
| $V_{S}$ |  |  |  |
| $V_{A B}$ |  |  |  |
| $V_{B C}$ |  |  |  |
| $V_{C D}$ |  |  |  |
| $V_{D E}$ |  |  |  |

Table 4.1 - Voltage Ladder Data

## Part V- Wheatstone Bridge

In this part of the lab, the GTA will assign you a value resistor $\mathbf{R x}_{\mathrm{x}}$. You must build and use a Wheatstone Bridge to verify the value of its resistance.


Figure 5.1 - Wheatstone Bridge

1. Build the Wheatstone Bridge in Figure 5.1. You will need the following components:

- $\mathrm{R}_{1}=\mathbf{2 k} \Omega, \mathrm{R}_{3}=5.1 \mathrm{k} \Omega, \mathrm{R}_{\mathbf{2}}=10 \mathrm{k} \Omega$ potentiometer
- DMM to measure voltage drop between nodes $A$ and $B\left(V_{A B}\right)$

2. After you have completed the setup, connect a 5 V supply forVs from the AD2.
3. Record the initial (unbalanced) value of $\mathbf{V}_{\mathrm{AB}}$ in Table 5.1.
4. Adjust the potentiometer until the bridge is balanced. (Hint: Refer to Part IV of the Prelab to recall what balanced means for a Wheatstone bridge circuit.)
a. Record $\mathrm{V}_{\mathrm{AB}}$ once the bridge is balanced in Table 5.1.
5. Remove the potentiometer from the circuit without adjusting it and use the DMM to measure its actual resistance.
a. Record the value of $R_{2}$ in Table 5.1.
6. Measure the exact resistances of $R_{1}$ and $R_{3}$ with the $D M M$.
7. Use the measured values for $R_{1}, R_{2}$, and $R_{3}$ and the expression you derived in Part IV of the Prelab to calculate the resistance of $\mathrm{Rx}_{\mathrm{x}}$. Show your calculations.
8. Remove $\mathbf{R x}_{\mathrm{x}}$ from the circuit and use the DMM to measure its exact resistance.
9. Compare your calculated and measured results for the resistance of $\mathbf{R x}_{\mathrm{x}}$ by calculating the percent error.

| Quantity | Measured | Calculated | Percent Error (\%) |
| :---: | :--- | :--- | :--- |
| $R_{X}$ |  |  |  |
| $V_{A B}$ (unbalanced) |  |  |  |
| $V_{A B}$ (balanced) |  |  | n/a |
| $R_{1}$ |  |  |  |
| $R_{2}$ |  |  |  |
| $R_{3}$ |  |  |  |

Table 5.1 - Wheatstone Bridge Data

## Post-Lab Analysis

For part 1 of the lab: Compare and contrast the calculated results from your prelab, to your Multisim simulations and finally to the DC measurements made in the lab itself. Show the percentage error in each case. Explain the differences between the calculated, measured, and simulated results then include a discussion on the reason for the discrepancies. Also explain the concept of tolerance in all the devices and equipment and how to compensate for the problem of inaccurate measurements.

For part 2 of the lab: Make sure to answer all questions asked in tandem with the lab instructions. Now that you have designed and observed the behavior of a voltage ladder / LED bar combination in detail, can you think of any applications for such a circuit? Name one or two possibilities in your report.

For part 3 of the Lab: Why does the bridge have to be 'balanced' in order to calculate the value of R2? Does your calculated value for $R_{2}$ (from step 3 c) match your measured value of $R_{2}$ (from step $3 d$ ), what is the $\%$ error? Research and explain several uses of the Wheatstonebridge.

