Knots in Washington (KiW) 49.75   April 22-24, 2022

List of Speakers and Abstracts

(1) **Plenary Speaker:** Colin C. Adams
Affiliation: Williams College
Co-authors: Jonah Greenberg, Kapoor Kabir, Yi Wang, Zhen Liang, Or Eisenberg, Kate O'Connor, Natalia Pacheco-Tallaj, Alexander Simons
In person
Title of talk: Hyperbolicity and Turaev Hyperbolicity of Classical and Virtual Knots
Abstract: All classical knots are either trivial, torus, satellite or hyperbolic. In the hyperbolic case, volume turns out to be a very useful invariant. We extend hyperbolicity to virtual knots, show that unlike the classical case, compositions of hyperbolic virtual knots are again hyperbolic and then introduce Turaev hyperbolicity and show every knot (even the trivial knot) is Turaev hyperbolic and hence has a Turaev hyperbolic volume.

(2) **Speaker:** Rhea Palak Bakshi
Affiliation: Institute for Theoretical Studies at ETH Zurich
Co-authors: Thang Lê and Jozef H. Przytycki
In person
Title of talk: Chebyshev representation of handle sliding relations in the KBSM
Abstract: Skein modules were introduced by Józef H. Przytycki as generalisations of the Jones and HOMFLYPT polynomial link invariants in the 3-sphere to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied of all. We show that the KBSM of the connected sum of two solid tori is isomorphic to the KBSM of a genus two handlebody modulo some specific handle sliding relations. Moreover, we show that these handle sliding relations can be written in terms of Chebyshev polynomials. This is joint work with Jozef H. Przytycki and Thang Lê.

(3) **Speaker:** Lizzie Buchanan
Advisor (if you are a graduate student): Vladimir Chernov
Affiliation: Dartmouth College
Co-authors:
In person
Title of talk: The Jones polynomial of a fibered positive link, and the search for knots with minimal almost-alternating diagrams
Abstract: We would ultimately like to produce an infinite family of knots that have a minimal (with respect to crossing number) almost-alternating diagram. While working on this problem, we found a new upper bound on the maximum degree of the Jones polynomial of a fibered positive link. In particular, the maximum degree of the Jones polynomial of a fibered positive knot is at most four times the minimum degree. With this result, we complete the classification of all knots of crossing number less than or equal to 12 as positive or not positive.

(4) **Speaker:** Kim Byeorhi
Affiliation: POSTECH, Rep. of Korea
Co-authors: Yongju Bae, Scott Carter
Online
Title of talk: On quandle extensions related to group extensions
Abstract: In this talk, we consider a quandle structure defined on a group G using an automorphism ϕ. If the automorphism is an inner automorphism by $\zeta \in G$, we denote this quandle as $(G, \triangleleft_\zeta)$. In this talk, we show a relationship between group extensions of a group G and quandle extensions of the quandle $(G, \triangleleft_\zeta)$. In fact, there exists a group homomorphism between the second cohomology groups of them. Next, we consider the quandle structure $(G, \triangleleft_\zeta)$ when G is an inner automorphism group of a quandle. Also, we introduce an interesting example about a relationship between extensions of a quandle and extensions of the inner automorphism group of the quandle.

(5) **Speaker:** Patricia Cahn
Affiliation: Smith College
Co-authors: Patricia Cahn, Elise Catania, Sarangoo Chimgee, Olivia Del Guercio, Jack Kendrick
Online
Title of talk: Dihedral Linking Invariants
Abstract: A Fox p-colored knot K in S^3 gives rise to a corresponding p-fold branched cover M of S^3 along K. The preimage of the knot K under the covering map is a (p + 1)/2 -component link L in M, and the set of pairwise linking numbers of the components of L is an invariant of K. This powerful invariant played a key role in the development of early knot tables, and appears in formulas for many other important knot and manifold invariants. We give an algorithm for computing this invariant for all odd p, generalizing an algorithm of Perko, and tabulate the invariant for thousands of p-colorable knots.

(6) **Speaker:** Daren Chen
Advisor (if you are a graduate student): Ciprian Manolescu
Affiliation: Stanford University
Co-authors:
Online
Title of talk: Twistings and the Alexander polynomial
Abstract: We give an explicit formula of the Alexander polynomial of the link obtained by adding an arbitrary number of full twists to positively oriented parallel n-strands in terms of the Alexander polynomials of the links obtained by adding $0, 1, \cdots, (n-1)$ full twists. From this, we see that the Alexander polynomials stabilize after adding sufficiently many full twists. The main tool used in the computation is expressing the Alexander polynomial using the vector space representation of $U_q(gl(1|1))$.

(7) **Speaker:** Vladimir Chernov
Affiliation: Dartmouth College
Co-authors Rustam Sadykov, KSU
Online
Title of talk: Some questions about virtual Legendrian knots
Abstract: Virtual Legendrian knots were introduced by Cahn and Levi and jointly with Sadykov we proved the Kuperberg type theorem for them. We will discuss a few open questions about the virtual Legendrian knots including the versions of the Ding-Geiges Theorem, Arnolds ́ 4 cusp conjecture and the applications of this to causality in spacetimes with the changing topology of the spacelike section in the spirit of our works with Nemirovski.

(8) **Speaker:** Paul Kainen
Advisor (if you are a graduate student):
Affiliation: Georgetown University
Co-authors: Richard Hammack, VIrginia Commonwealth University
In person
Title of talk: Euler ́s Theorem for regular CW-complexes
Abstract: Klee proved that the dual of an n-dimensional pseudomanifold is (n+1)-connected. But he assumed a simplicial complex. We prove that if one only has a regular CW-complex, the dual is bridgeless - i.e., has no separating edges. This enables us to extend Euler ́s Theorem from 2-d (Math Magazine, 2022/2023) to n-dimensions.

(9) **Plenary Speaker:** Uwe Kaiser
Affiliation: Boise State University
Co-authors:
In person
Title of talk: Link homotopy skein modules and the fundamental group of 3-manifolds
Abstract: Link homotopy skein modules of oriented 3-manifolds have been introduced by Jim Hoste and Jozef Przytycki in 1990 and Jozef Przytycki in 1991. I will survey the known results on link homotopy skein modules (mostly proven in the years 2000-2003), and in particular discuss the

relation with string topology and the conjugacy problem in the fundamental groups of 3-manifolds. It is an interesting and still not completely understood problem to describe link homotopy skein modules of connected sums and unions along tori (JSJ-decomposition). Also, the problem of understanding what link homotopy invariants for links in 3-manifolds are detected in the modules will be discussed. We will present link homotopy skein modules as an interesting object between the fundamental group of a 3-manifold and the usually considered isotopy skein modules. The fact that the relations are not local (the skein relations are applied only to crossings of distinct components) has the consequence that many powerful techniques recently developed for the understanding of skein modules do not apply in this setting.

(10) **Plenary Speaker:** Louis H. Kauffman
Affiliation:
Co-authors: Neslihan Gugumcu
On line
Title of talk: A State Sum Invariant for Knotoids
Abstract: This talk is joint work with Neslihan Gugumcu. Recall that a knotoid diagram is a knot diagram with two free ends and that the ends can be in distinct regions of the diagram. Equivalence for knotoid diagrams is generated by Reidemeister moves that do not shift an arc across the endpoints. Thus knotoids are a generalization of 1-1 tangles. The terminology knotoid and many basic results about them are due to Tuarev. Kauffman and Gugumcu have previously used techniques in virtual knot theory to study knotoids. In this talk we return to the classical Alexander polynomial. We generalize the combinatorial structure underlying the Formal Knot Theory state summation for the Alexander-Conway polynomial to a potential function P[k] for knotoids k. This new invariant is obtained by summing over all connected smoothings of the knotoid diagram. We show that the invariant can detect mirror images and reversals for some knotoids and ask other questions about its structure. The states have structure similar to the FKT states, and we prove a Clock Theorem for them. The talk will discuss the possible use of these states to categorify the invariant P[K] along the lines of combinatorial Heegaard-Floer homology.

(11) **Plenary Speaker:** Mikhail Khovanov
Affiliation: Columbia University
Co-authors:
In person
Title of talk: Facets of universal construction
Abstract: Universal construction takes a multiplicative invariant of closed n-manifolds and builds a topological theory for n-dimensional cobordisms between closed (n-1)-manifolds. We'll review examples and uses of universal construction, including in link homology and for decorated two- and one-dimensional manifolds, where it relates to generalized Deligne categories, biadjoint functors, noncommutative rational power series and automata.

(12) **Plenary Speaker:** Mee Seong Im
Affiliation: United States Naval Academy
Co-authors: Mikhail Khovanov
In person
Title of talk: One-dimensional topological theories with decorations and finite state automata
Abstract: I will explain a surprising relation between topological theories for one-dimensional manifolds with zero-dimensional defects and values in the Boolean semiring and finite-state automata and their generalizations. This is joint with Mikhail Khovanov.

(13) **Speaker:** Christine Ruey Shan Lee
Affiliation: University of South Alabama
Co-authors: Carmen Caprau, Nicolle Gonzalez, Radmila Sazdanovic, Melissa Zhang
In person
Title of talk: Full twists and periodicity of categorified Jones-Wenzl projectors
Abstract: It is known that the limiting complex of infinitely many full twists on n strands exists by the work of Stosic and recovers the categorified Jones-Wenzl projector by the work of Rozansky. However, the rate of convergence and relationship between the two chain complexes remain not well understood. In this talk, I will discuss some results to clarify this relationship for 3-braids.

(14) **Distinguished Graduate Student Speaker:** Ryan Maguire
Advisor (if you are a graduate student): Vladimir Chernov
Affiliation: Dartmouth College
Co-authors:
Online
Title of talk: Conjectures on the Khovanov and Knot Floer Homologies of Legendrian and Transversally Simple Knots
Abstract: A theorem of Kronheimer and Mrowka states that Khovanov homology is able to detect the unknot. That is, if a knot has the Khovanov homology of the unknot, then it is equivalent to it. A similar result holds for Knot Floer homology. The unknot is the simplest of the torus knots, which is a class of knots known to be Legendrian simple. It is conjectured that Khovanov and Knot Floer homology are able to distinguish Legendrian and Transversally simple knots. Numerical evidence is provided for all knots up to and including 17 crossings.

(15) **Speaker:** Maciej Markiewicz
Advisor (if you are a graduate student): Maciej Borodzik
Affiliation: University of Warsaw
Co-authors: David Cimasoni, Wojciech Politarczyk and Gaetan Simian
Online
Title of talk: Torres formula for multivariable link signature
Abstract: Multivariable signature is an invariant of colored links, defined as a certain function from an open cube to integers. It can be defined as a signature of an evaluation of a matrix associated to the link, the determinant of which is the multivariable Alexander polynomial of the link. This suggests that the signature function can be extended to the multidimensional torus and the values at the boundary should be closely related to the value of the signature for a suitable sublink, as is the case for the Alexander polynomial by a formula of Torres. Our work aims to make this connection precise and to investigate the four-dimensional information contained in this relation.

(16) **Speaker:** Daniel Matthews
Affiliation: Monash University
Co-authors: Jessica Purcell
Online
Title of talk: A symplectic basis for 3-manifold triangulations
Abstract: Neumann and Zagier in the 1980s introduced a symplectic vector space associated to an ideal triangulation of a cusped 3-manifold, such as a knot complement. We will discuss an interpretation for this symplectic structure in terms of the topology of the 3-manifold. This joint work with Jessica Purcell involves train tracks, Heegaard splittings, and is related to Ptolemy varieties, geometric quantisation, and the A-polynomial.

(17) **Speaker:** Seppo Niemi-Colivn
Advisor (if you are a graduate student): Adam Levine
Affiliation: Duke University
Co-authors:
Online
Title of talk: Invariance of Knot Lattice Homology and Homotopy
Abstract: Links of singularity and generalized algebraic links are ways of constructing three-manifolds and smooth links inside them from complex algebraic surfaces and complex curves inside them. Némethi created lattice homology as an invariant for links of normal surface singularities which developed out of computations for Heegaard Floer homology and also has a homotopy theoretic interpretation. Later Ozsváth, Stipsicz, and Szabó defined knot lattice homology for generalized algebraic knots in rational homology spheres, which is known to play a similar role to knot Floer homology. I discuss a proof that knot lattice homology and homotopy are invariants of the knot type, which had been previously suspected but not confirmed.

(18) **Speaker:** Sam Nelson
Affiliation: Claremont McKenna College
In person
Title of talk: Experimental Knot Music
Abstract: In this talk I will describe my recent project of composing music by sequencing elements of homsets from algebraic structures associated to knots to finite structures in the same category, e.g. biquandles or Niebrzydowski tribrackets. If time permits I will illustrate the process by composing a song from knot homsets live at the end of the talk.

(19) **Speaker:** Ken Perko
Affiliation: N/A
In person
Title of talk: Looking at Linking Numbers
Abstract: We shall show how linking numbers between branch curves of 3- and 4-fold simple covering spaces of knots are affected by crossing reversals.

(20) **Plenary Speaker:** Jessica Purcell
Affiliation: Monash University
Co-authors: Josh Howie and Effie Kalfagianni
Online
Title of talk: Geometry of alternating links on surfaces
Abstract: It is typically hard to relate the geometry of a knot complement to a diagram of the knot, but over many years mathematicians have been able to relate geometric properties of classical alternating knots to their diagrams. Recently, Howie and I have modified these techniques to investigate geometry of a much wider class of knots, namely alternating knots with diagrams on general surfaces embedded in general 3-manifolds. This has resulted in lower bounds on volumes, information on the geometry of checkerboard surfaces, restrictions on exceptional Dehn fillings, and other geometric properties. However, we were unable to extend upper volume bounds broadly. In fact, recently Kalfagianni and I showed that an upper bound must depend on the 3-manifold in which the knot is embedded: We find upper bounds for virtual knots, but not for other families. In this talk, I will discuss this work, and some remaining open questions.

(21) **Speaker:** Prasad Senesi
Affiliation: The Catholic University of America
Co-authors: Mohamed Elhamdadi, Emanuele Zappala
In person
Title of talk: Quandle representation theory: dihedral and cyclic
Abstract: This talk will summarize recent research in the representation theory of finite quandles; that is, for a finite quandle Q, we investigate quandle homomorphisms from Q to Aut(V), where V is a finite-dimensional complex vector space V, and the corresponding invariant subspaces of V. In particular, we will discuss results for the dihedral quandles and for cyclic quandles. In an effort to study their representations, we provide a full classification for these cyclic quandles, building upon earlier results of Kamada, Tamaru, Wada, and Vendramin. This is joint work with Mohamed Elhamdadi and Emanuele Zappala.

(22) **Plenary Speaker:** Marithania Silvero
Affiliation: University of Seville
Co-authors: Józef H. Przytycki
In person
Title of talk: Computing Khovanov homotopy type of 4-braids in polynomial time
Abstract: Khovanov homology is a link invariant which generalizes Jones polynomial. In general, computing Jones polynomial (so also Khovanov homology) is NP-hard. However, if we consider a closed braid of a fixed number of strands, it is well-known that all classical quantum invariants (in particular Jones polynomial) can be computed in polynomial time. We conjecture that the complexity of computing Khovanov homology of a closed braid of fixed number of strands is polynomial with respect to the number of crossings.

In this talk we show some advances on the conjecture, showing that the result holds when considering extreme Khovanov homology of closed braids of at most 4 strands. As a consequence, we get an obstruction for a link to have braid index 4 in terms of its extreme Khovanov homology.

(23) **Speaker:** Keshav Srinivasan
Advisor (if you are a graduate student): Valentina Harizanov
Affiliation: George Washington University
Co-authors:
In person
Title of talk: Effective Ultrapowers of Graphs
Abstract: While typical ultrapower constructions in mathematical logic produce uncountable models, effective ultrapower constructions allow us to algorithmically build countable non-standard models with interesting properties. I will explain how effective ultrapowers work, and outline recent research I've done on effective ultrapowers for structures from various important classes of directed graphs. I will state a universal embeddability result for graphs in effective ultrapowers of strongly locally finite graphs. In general, by Dimitrov's theorem, only first-order properties expressed by sentences at lower levels of arithmetical hierarchy are preserved by effective ultrapowers. I will outline a classification of certain structures from concrete classes when the effective ultrapower is isomorphic to the original structure.

(24) **Speaker:** Emanuele Zappala
Affiliation: Yale University
Co-authors: Viktor Abramov, Mohamed Elhamdadi
Online
Title of talk: n-Lie algebras, their cohomology and the Yang-Baxter equation
Abstract: In this presentation I will talk about certain generalizations of quandles, called n-ary self-distributive objects, generated by n-ary Lie algebras. In particular, I will show that n-Lie algebras determine solutions to the Yang-Baxter (YB) equation that are generally nontrivial. Moreover, the associated self-distributive objects admit a cohomology theory that generalizes (in some sense) the usual cohomology of quandles, and the second cohomology group can be used to obtain deformations of the associated YB operators. I will mention, also, that the cohomology of Lie algebras is isomorphic to the self-distributive one, under suitable additional hypotheses, showing that the traditional deformation theory of Gerstenhaber-Schack (and Nijenhuis-Richardson) can be applied to produce deformations of the aforementioned YB operators. This translates into an injection from n-ary

Lie cohomology into the YB cohomology of the associated operators. I will conclude by discussing some applications to link invariants.