Online Appendix A: summary of notation

Panels A, B, and F show the notation for statistics of, respectively, assets, portfolios, and optimal portfolios. Panels C, D, and E show the notation for, respectively, the M-V, M-SCVaR, and SCVaR-constrained M-V frontiers. Part I of these three panels considers the case where a risk-free asset is absent, whereas part II of such panels considers the case where it is present. Panel G provides formulas for the relative changes in various statistics of the optimal portfolios due to an SCVaR constraint. While part I of this panel considers the case where the Volcker rule is present, part II considers the case where it is absent. Panel H provides formulas for the relative changes in various statistics of the optimal portfolios due to the Volcker rule. While part I of this panel considers the case where an SCVaR constraint is present, part II considers the case where it is absent. Panel I explains how we assess the impact of an SCVaR constraint and the Volcker rule on various statistics of the optimal portfolios.

Panel A. Statistics of assets

$\boldsymbol{\mu}$	Vector of expected risky asset returns
r_f	Return of risk-free asset
$\boldsymbol{\Sigma}$	Matrix of variances and covariances of risky asset returns
$\boldsymbol{\mu}_s$	Vector of stressed expected risky asset returns
$r_{f,s}$	Stressed return of risk-free asset
$\boldsymbol{\Sigma}_s$	Matrix of stressed variances and covariances of risky asset returns
$\boldsymbol{\Sigma}_\varphi$	Weighted average of matrices $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}_s$ with respective weights of $1-\varphi$ and φ

Panel B. Statistics of portfolios

$(E[r_{\boldsymbol{w}}], \sigma[r_{\boldsymbol{w}}])$	Expected return and SD of portfolio $\boldsymbol{w}$
$C_\alpha[r_{\boldsymbol{w}}]$	CVaR at confidence level α of portfolio $\boldsymbol{w}$
$(E_s[r_{\boldsymbol{w}}], \sigma_s[r_{\boldsymbol{w}}])$	Stressed expected return and stressed SD of portfolio $\boldsymbol{w}$
$C_{s,\alpha}[r_{\boldsymbol{w}}]$	SCVaR at confidence level α of portfolio $\boldsymbol{w}$

Panel C. M-V frontier

Part I: Risk-free asset absent	
$\boldsymbol{w}_E$	Portfolio on the M-V frontier with an expected return of E
$(\boldsymbol{w}_0, \boldsymbol{w}_1)$	Portfolios that span $\boldsymbol{w}_E$
$(\theta_E, 1-\theta_E)$	Weights of portfolios that span $\boldsymbol{w}_E$
Part II: Risk-free asset present	
$\overline{\boldsymbol{w}}_E$	Portfolio on the M-V frontier with an expected return of E
$(\overline{\boldsymbol{w}}_0, \overline{\boldsymbol{w}}_1)$	Portfolios that span $\overline{\boldsymbol{w}}_E$
$(\overline{\theta}_E, 1-\overline{\theta}_E)$	Weights of portfolios that span $\overline{\boldsymbol{w}}_E$

Panel D. M-SCVaR frontier

Part I: Risk-free asset absent	
$\boldsymbol{w}_{\alpha,E}$	Portfolio on the M-SCVaR frontier with a confidence level of α and an expected return of E
$(\boldsymbol{w}_{0,s}, \boldsymbol{w}_{1,s}, \boldsymbol{w}_{2,s})$	Portfolios that span portfolio $\boldsymbol{w}_{\alpha,E}$
$(\theta_{0,\alpha,E}, \theta_{1,\alpha,E}, 1-\theta_{0,\alpha,E}-\theta_{1,\alpha,E})$	Weights of portfolios that span $\boldsymbol{w}_{\alpha,E}$
Part II: Risk-free asset present	
$\overline{\boldsymbol{w}}_{\alpha,E}$	Portfolio on the M-SCVaR frontier with a confidence level of α and an expected return of E
$(\overline{\boldsymbol{w}}_{0}, \overline{\boldsymbol{w}}_{1,s}, \overline{\boldsymbol{w}}_{2,s})$	Portfolios that span $\overline{\boldsymbol{w}}_{\alpha,E}$
$(\overline{\theta}_{0,\alpha,E}, \overline{\theta}_{1,\alpha,E}, 1-\overline{\theta}_{0,\alpha,E}-\overline{\theta}_{1,\alpha,E})$	Weights of portfolios that span $\overline{\boldsymbol{w}}_{\alpha,E}$

Panel E. SCVaR-constrained M-V frontier

Part I: Risk-free asset absent	
$\boldsymbol{w}_{\alpha,C_s,E}$	Portfolio on the SCVaR-constrained M-V frontier with a confidence level of α, a bound of C_s, and an expected return of E
$(\boldsymbol{w}_{0,\varphi_{\alpha,C_s,E}}, \boldsymbol{w}_{1,\varphi_{\alpha,C_s,E}}, \boldsymbol{w}_{2,\varphi_{\alpha,C_s,E}})$	Portfolios that span $\boldsymbol{w}_{\alpha,C_s,E}$ when the SCVaR constraint binds
$(\theta_{0,\alpha,C_s,E}, \theta_{1,\alpha,C_s,E}, 1-\theta_{0,\alpha,C_s,E}-\theta_{1,\alpha,C_s,E})$	Weights of portfolios that span $\boldsymbol{w}_{\alpha,C_s,E}$
Part II: Risk-free asset present	
$\overline{\boldsymbol{w}}_{\alpha,C_s,E}$	Portfolio on the SCVaR-constrained M-V frontier with a confidence level of α, a bound of C_s, and an expected return of E
$(\overline{\boldsymbol{w}}_{0}, \overline{\boldsymbol{w}}_{1,\overline{\varphi}_{\alpha,C_s,E}}, \overline{\boldsymbol{w}}_{2,\overline{\varphi}_{\alpha,C_s,E}})$	Portfolios that span $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$ when the SCVaR constraint binds
$(\overline{\theta}_{0,\alpha,C_s,E}, \overline{\theta}_{1,\alpha,C_s,E}, 1-\overline{\theta}_{0,\alpha,C_s,E}-\overline{\theta}_{1,\alpha,C_s,E})$	Weights of portfolios that span $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$

Panel F. Statistics of optimal portfolios[A.1]

SD, SCVaR at confidence level α, and CER of the optimal portfolio	SCVaR constraint	Volcker rule
$(\sigma^A, C^A_{s,\alpha}, CER^A)$	Absent	Present
$(\sigma^P, C^P_{s,\alpha}, CER^P)$	Present	Present
$(\sigma^B, C^B_{s,\alpha}, CER^B)$	Absent	Absent
$(\sigma^Q, C^Q_{s,\alpha}, CER^Q)$	Present	Absent

Panel G. Relative changes in various statistics of the optimal portfolios due to an SCVaR constraint

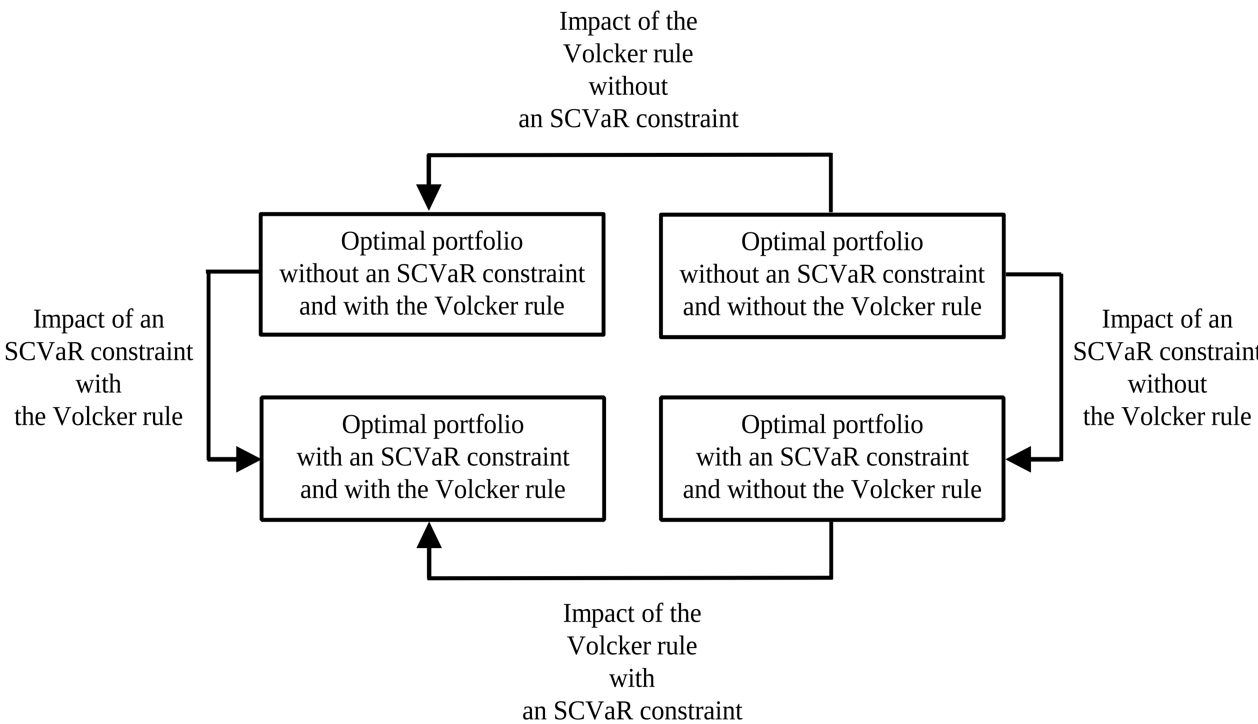
Part I: Volcker rule present	
Relative change in SD	$\frac{\sigma^P}{\sigma^A} - 1$
Relative change in SCVaR	$\frac{C^P_{s,\alpha}}{C^A_{s,\alpha}} - 1$
Relative change in SD-to-SCVaR ratio	$\frac{\sigma^P/C^P_{s,\alpha}}{\sigma^A/C^A_{s,\alpha}} - 1$
Relative change in CER	$\frac{CER^P}{CER^A} - 1$
Part II: Volcker rule absent	
Relative change in SD	$\frac{\sigma^Q}{\sigma^B} - 1$
Relative change in SCVaR	$\frac{C^Q_{s,\alpha}}{C^B_{s,\alpha}} - 1$
Relative change in SD-to-SCVaR ratio	$\frac{\sigma^Q/C^Q_{s,\alpha}}{\sigma^B/C^B_{s,\alpha}} - 1$
Relative change in CER	$\frac{CER^Q}{CER^B} - 1$

Panel H. Relative changes in various statistics of the optimal portfolios due to the Volcker rule

Part I: SCVaR constraint present	
Relative change in SD	$\frac{\sigma^P}{\sigma^Q} - 1$
Relative change in SCVaR	$\frac{C^P_{s,\alpha}}{C^Q_{s,\alpha}} - 1$
Relative change in SD-to-SCVaR ratio	$\frac{\sigma^P/C^P_{s,\alpha}}{\sigma^Q/C^Q_{s,\alpha}} - 1$
Relative change in CER	$\frac{CER^P}{CER^Q} - 1$
Part II: SCVaR constraint absent	
Relative change in SD	$\frac{\sigma^A}{\sigma^B} - 1$
Relative change in SCVaR	$\frac{C^A_{s,\alpha}}{C^B_{s,\alpha}} - 1$
Relative change in SD-to-SCVaR ratio	$\frac{\sigma^A/C^A_{s,\alpha}}{\sigma^B/C^B_{s,\alpha}} - 1$
Relative change in CER	$\frac{CER^A}{CER^B} - 1$

[A.1] SCVaR is given by: (i) Eq. (4) if the Volcker rule is present; and (ii) Eq. (13) if it is absent.

Panel I. Assessing the impact of an SCVaR constraint and the Volcker rule on various statistics of the optimal portfolios



Online Appendix B: summary of results

Panel A summarizes how the size of the bound affects the location of the portfolio on the SCVaR-constrained M-V frontier with a given expected return. Panels B, C, and D summarize the impact of, respectively, an SCVaR constraint, the Volcker rule, and an SD constraint on various statistics of the optimal portfolios in our example.

Panel A. The size of the bound and the location of a portfolio on the SCVaR-constrained M-V frontier with a given expected return

Size of bound	Portfolio on the SCVaR-constrained M-V frontier with a given expected return
Smaller than the SCVaR of the portfolio on the M-SCVaR frontier with the same expected return	Does not exist
Equal to the SCVaR of the portfolio on the M-SCVaR frontier with the same expected return	Is on the M-SCVaR frontier
Strictly between the SCVaRs of the portfolios on the M-SCVaR and M-V frontiers with the same expected return	Lies strictly between the M-SCVaR and M-V frontiers
Equal to or larger than the SCVaR of the portfolio on the M-V frontier with the same expected return	Is on the M-V frontier

Panel B. Impact of an SCVaR constraint on various statistics of the optimal portfolios[B.1]

Relative change in:	Impact of an SCVaR constraint	Effect of using a larger: Risk aversion coefficient	Effect of using a larger: Bound
Part I: Volcker rule present			
SD	Negative	Less negative	
SCVaR	Negative	Less negative	
SD-to-SCVaR ratio	Positive	Less positive	
CER	Negative	Less negative	
Part II: Volcker rule absent			
SD	Negative	Less negative	
SCVaR	Negative	Less negative	
SD-to-SCVaR ratio	Positive	Less positive	
CER	Negative	Less negative	

Panel C. Impact of the Volcker rule on various statistics of the optimal portfolios

Relative change in:	Impact of the Volcker rule	Effect of using a larger: Risk aversion coefficient	Effect of using a larger: Bound
Part I: SCVaR constraint present			
SD	Depends[B.2]	Depends	
SCVaR	Zero[B.3]	No effect	
SD-to-SCVaR ratio	Depends	Depends	
CER	Negative	Depends	
Part II: SCVaR constraint absent			
SD	Negative	No effect	–
SCVaR	Negative	No effect	–
SD-to-SCVaR ratio	Roughly zero	No effect	–
CER	Negative	Slightly less negative	–

[B.1] SCVaR is given by: (i) Eq. (4) if the Volcker rule is present; and (ii) Eq. (13) if it is absent. This remark applies also to panel C.

[B.2] Here, the term 'depends' means that the sign of the relative change in the SD of the optimal portfolio due to the Volcker rule depends on the size of the risk aversion coefficient and bound. A similar remark applies to subsequent panel cells where this term appears.

[B.3] The relative change in SCVaR is zero because the SCVaR constraint binds with and without the Volcker rule (for all the values of the risk aversion coefficient and bound used in our example).

Panel D. Impact of an SD constraint on various statistics of the optimal portfolios

<table>
<tr><th rowspan="2">Relative change in:</th><th rowspan="2">Impact of an SD constraint[B.4]</th><th colspan="2">Effect of using a larger:</th></tr>
<tr><th>Risk aversion coefficient</th><th>Bound</th></tr>
<tr><td colspan="4">Part I: Volcker rule present</td></tr>
<tr><td>SD</td><td>Negative</td><td colspan="2">Less negative</td></tr>
<tr><td>SCVaR</td><td>Negative</td><td colspan="2">Less negative</td></tr>
<tr><td>CER</td><td>Negative</td><td colspan="2">Less negative</td></tr>
<tr><td colspan="4">Part II: Volcker rule absent</td></tr>
<tr><td>SD</td><td>Negative</td><td colspan="2">Less negative</td></tr>
<tr><td>SCVaR</td><td>Negative</td><td colspan="2">Less negative</td></tr>
<tr><td>CER</td><td>Negative</td><td colspan="2">Less negative</td></tr>
</table>

[B.4] An SD constraint does not affect the risk-to-minimum capital requirement ratio of the optimal portfolio if risk in measured by SD and minimum capital requirements are proportional to SDs.

Online Appendix C: proofs of theoretical results

Definition of $\theta_{0,\alpha,E}$ and $\theta_{1,\alpha,E}$. In Eq. (5), the weights of $\boldsymbol{w}_{0,s}$ and $\boldsymbol{w}_{1,s}$ in $\boldsymbol{w}_{\alpha,E}$ are, respectively:

$$\theta_{0,\alpha,E} \equiv \frac{c_s}{b_s c_s - a_s^2}\left[(b_s - a_s E) + (a_s f_s - b_s d_s)\sqrt{\frac{h_s}{y_\alpha^2 - g_s}}\right]$$

and:

$$\theta_{1,\alpha,E} \equiv \frac{a_s}{b_s c_s - a_s^2}\left[(c_s E - a_s) + (a_s d_s - c_s f_s)\sqrt{\frac{h_s}{y_\alpha^2 - g_s}}\right]$$

where $h_s \equiv \frac{c_s E^2 - 2a_s E + b_s}{b_s c_s - a_s^2}$ is a positive number.

Definition of $\overline{\theta}_{0,\alpha,E}$ and $\overline{\theta}_{1,\alpha,E}$. In Eq. (D.2), the weights of $\overline{\boldsymbol{w}}_0$ and $\overline{\boldsymbol{w}}_{1,s}$ in $\overline{\boldsymbol{w}}_{\alpha,E}$ are, respectively:

$$\overline{\theta}_{0,\alpha,E} \equiv 1 - \frac{(a_s - c_s r_f)}{j_s}\left[(E - r_f) - k_s\sqrt{\frac{\frac{1}{j_s}(E - r_f)^2}{y_\alpha^2 - \overline{g}_s}}\right] - (d_s - c_s r_{f,s})\sqrt{\frac{\frac{1}{j_s}(E - r_f)^2}{y_\alpha^2 - \overline{g}_s}}$$

and:

$$\overline{\theta}_{1,\alpha,E} \equiv \frac{(a_s - c_s r_f)}{j_s}\left[(E - r_f) - k_s\sqrt{\frac{\frac{1}{j_s}(E - r_f)^2}{y_\alpha^2 - \overline{g}_s}}\right].$$

Proof of Theorem 1. Suppose that $\alpha > \alpha_s$, $C_s \in \mathbb{R}$, and $E \in \mathbb{R}$. First, we show (i). Assume that $C_s < C_{s,\alpha,\boldsymbol{w}_{\alpha,E}}$. Since $\boldsymbol{w}_{\alpha,E}$ is on the M-SCVaR frontier and $C_s < C_{s,\alpha,\boldsymbol{w}_{\alpha,E}}$, no portfolio with an expected return of E meets the SCVaR constraint. Hence, no portfolio exists on the SCVaR-constrained M-V frontier with an expected return of E. This completes the first part of our proof.

Second, we show (ii). Assume that $C_s = C_{s,\alpha,\boldsymbol{w}_{\alpha,E}}$. Since $\boldsymbol{w}_{\alpha,E}$ is on the M-SCVaR frontier, $\boldsymbol{w}_{\alpha,E}$ is the unique portfolio with an expected return of E that meets the SCVaR constraint. Therefore, Eq. (8) holds. This completes the second part of our proof.

Third, we show (iii). Assume that $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C < C_{s,\alpha,\boldsymbol{w}_E}$. Note that $\boldsymbol{w}^* \equiv \boldsymbol{w}_{\alpha,C_s,E}$ solves:[C.1]

$$\min_{\boldsymbol{w}\in\mathbb{R}^N} \quad \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \tag{C.1}$$

$$s.t. \qquad \boldsymbol{w}'\boldsymbol{1} = 1 \tag{C.2}$$

$$\boldsymbol{w}'\boldsymbol{\mu} = E \tag{C.3}$$

$$y_\alpha\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}_s\boldsymbol{w}} - \boldsymbol{w}'\boldsymbol{\mu}_s \leq C_s. \tag{C.4}$$

A first-order condition for $\boldsymbol{w}^*$ to solve problem (C.1) subject to constraints (C.2)–(C.4) is:

$$\boldsymbol{\Sigma}\boldsymbol{w}^* + \lambda_1\boldsymbol{1} + \lambda_2\boldsymbol{\mu} + \lambda_3\left[\frac{y_\alpha\boldsymbol{\Sigma}_s\boldsymbol{w}^*}{\sqrt{(\boldsymbol{w}^*)'\boldsymbol{\Sigma}_s\boldsymbol{w}^*}} - \boldsymbol{\mu}_s\right] = \boldsymbol{0} \tag{C.5}$$

[C.1] The existence of a solution to this problem follows from: (i) the function $F : \mathbb{R}^N \to \mathbb{R}$ defined by $F(\boldsymbol{w}) = \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}$ is continuous and coercive; (ii) the set of vectors in $\mathbb{R}^N$ that meet constraints (C.2)–(C.4) is non-empty and closed; and (iii) a continuous and coercive function has a minimum over a non-empty closed set. The uniqueness of this solution follows from: (a) the strict convexity of function F; and (b) and the convexity of the set of vectors in $\mathbb{R}^N$ that meet constraints (C.2)–(C.4). For a discussion of properties of coercive functions, see Beck (2014, Ch. 2).

where $\lambda_1 \in \mathbb{R}$, $\lambda_2 \in \mathbb{R}$, and $\lambda_3 \in \mathbb{R}_+$ are multipliers associated with these constraints.[C.2] Since $C_s < C_{s,\alpha,\boldsymbol{w}_E}$, $\boldsymbol{w}^*$ is not on the M-V frontier. Hence, $\lambda_3 > 0$. Let:

$$\eta \equiv \lambda_3 \left[\frac{y_\alpha}{\sqrt{(\boldsymbol{w}^*)'\boldsymbol{\Sigma}_s \boldsymbol{w}^*}} \right]. \tag{C.6}$$

Since $y_\alpha > 0$, $\lambda_3 > 0$, and $(\boldsymbol{w}^*)'\boldsymbol{\Sigma}_s \boldsymbol{w}^* > 0$, we have $\eta > 0$. Eqs. (C.5) and (C.6) imply that:

$$(\boldsymbol{\Sigma} + \eta \boldsymbol{\Sigma}_s)\, \boldsymbol{w}^* = -\lambda_1 \mathbf{1} - \lambda_2 \boldsymbol{\mu} + \lambda_3 \boldsymbol{\mu}_s. \tag{C.7}$$

Let:

$$\varphi^* \equiv \frac{\eta}{1+\eta}. \tag{C.8}$$

Since $\eta > 0$, Eq. (C.8) implies that $\varphi^* \in (0,1)$. Dividing both sides of Eq. (C.7) by $(1+\eta)$ and using Eq. (C.8), we have:

$$\boldsymbol{\Sigma}_{\varphi^*} \boldsymbol{w}^* = -\left(\frac{\lambda_1}{1+\eta}\right) \mathbf{1} - \left(\frac{\lambda_2}{1+\eta}\right) \boldsymbol{\mu} + \left(\frac{\lambda_3}{1+\eta}\right) \boldsymbol{\mu}_s. \tag{C.9}$$

Premultiplying Eq. (C.9) by $\boldsymbol{\Sigma}_{\varphi^*}^{-1}$, we obtain:

$$\boldsymbol{w}^* = -\left(\frac{\lambda_1}{1+\eta}\right) (\boldsymbol{\Sigma}_{\varphi^*}^{-1} \mathbf{1}) - \left(\frac{\lambda_2}{1+\eta}\right) (\boldsymbol{\Sigma}_{\varphi^*}^{-1} \boldsymbol{\mu}) + \left(\frac{\lambda_3}{1+\eta}\right) (\boldsymbol{\Sigma}_{\varphi^*}^{-1} \boldsymbol{\mu}_s). \tag{C.10}$$

Let:

$$\theta_{0,\alpha,C_s,E} \equiv -\left(\frac{\lambda_1}{1+\eta}\right) c_{\varphi^*} \tag{C.11}$$

and:

$$\theta_{1,\alpha,C_s,E} \equiv -\left(\frac{\lambda_2}{1+\eta}\right) a_{\varphi^*}. \tag{C.12}$$

Eqs. (C.10)–(C.12) imply that Eq. (9) holds with $\varphi_{\alpha,C_s,E} = \varphi^*$. This completes the third part of our proof.

Fourth, we show (iv). Assume that $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$. Since $\boldsymbol{w}_E$ is on the M-V frontier and meets the SCVaR constraint, Eq. (10) holds. This completes the fourth part of our proof. ■

A numerical approach to find $\varphi_{\alpha,C_s,E}$. Suppose that $\alpha > \alpha_s$, $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C_s < C_{s,\alpha,\boldsymbol{w}_E}$, and $E \in \mathbb{R}$. The following notation is useful. Let $\boldsymbol{\Psi} \equiv [\mathbf{1} \;\; \boldsymbol{\mu} \;\; \boldsymbol{\mu}_s]$ denote a $N \times 3$ matrix. For any $E_s \in \mathbb{R}$, let $\boldsymbol{\kappa}_{E_s} \equiv [1 \;\; E \;\; E_s]'$ denote a 3×1. For any $\varphi \in (0,1)$, let $\boldsymbol{\Upsilon}_\varphi \equiv (\boldsymbol{\Psi}'\boldsymbol{\Sigma}_\varphi^{-1}\boldsymbol{\Psi})^{-1}$ denote a 3×3 matrix. For any $(m_1, m_2) \in \{1,2,3\} \times \{1,2,3\}$, let $\Upsilon_{\varphi,m_1,m_2}$ denote the element in row m_1 and column m_2 of $\boldsymbol{\Upsilon}_\varphi$. Let $E_s^* \equiv E_s[r_{\boldsymbol{w}^*}]$ and $\sigma_s^* \equiv \sigma_s[r_{\boldsymbol{w}^*}]$ where $\boldsymbol{w}^*$ is defined in the proof of Theorem 1. Also, let $\boldsymbol{\pi}^* \equiv [\pi_1^* \;\; \pi_2^* \;\; \pi_3^*]'$ where $\pi_1^* \equiv -\frac{\lambda_1}{1+\eta}$, $\pi_2^* \equiv -\frac{\lambda_2}{1+\eta}$, and $\pi_3^* \equiv \frac{\lambda_3}{1+\eta}$. Here, λ_1, λ_2, λ_3, and η are defined in the proof of Theorem 1.

Using Eq. (C.10), we have:

$$\boldsymbol{w}^* = (\boldsymbol{\Sigma}_{\varphi^*}^{-1}\boldsymbol{\Psi})\boldsymbol{\pi}^*. \tag{C.13}$$

Premultiplying Eq. (C.13) with $\boldsymbol{\Psi}'$, we obtain:

$$\boldsymbol{\kappa}_{E_s^*} = (\boldsymbol{\Psi}'\boldsymbol{\Sigma}_{\varphi^*}^{-1}\boldsymbol{\Psi})\boldsymbol{\pi}^*. \tag{C.14}$$

Premultiplying Eq. (C.14) by $\boldsymbol{\Upsilon}_{\varphi^*} = (\boldsymbol{\Psi}'\boldsymbol{\Sigma}_{\varphi^*}^{-1}\boldsymbol{\Psi})^{-1}$, we have:

$$\boldsymbol{\pi}^* = \boldsymbol{\Upsilon}_{\varphi^*}\boldsymbol{\kappa}_{E_s^*}. \tag{C.15}$$

[C.2] Note that the Kuhn-Tucker conditions are necessary and sufficient for $\boldsymbol{w}^*$ to solve the problem. For a discussion of such conditions, see Horst, Pardalos, and Thoai (2000, Ch. 1).

Since the SCVaR constraint binds:

$$E_s^* = y_\alpha \sigma_s^* - C_s. \tag{C.16}$$

It follows from Eqs. (C.15) and (C.16) as well as the definition of $\boldsymbol{\kappa}_{E_s^*}$ that:

$$\pi_3^* = \Upsilon_{\varphi^*,3,1} + \Upsilon_{\varphi^*,3,2} E + \Upsilon_{\varphi^*,3,3} \left(y_\alpha \sigma_s^* - C_s \right). \tag{C.17}$$

The definition of π_3^* as well as Eqs. (C.6) and (C.8) imply that:

$$\pi_3^* = (\varphi^*/y_\alpha)\sigma_s^*. \tag{C.18}$$

Noting that $(\varphi^*/y_\alpha - \Upsilon_{\varphi^*,3,3} y_\alpha) \neq 0$, Eqs. (C.17) and (C.18) imply that:[C.3]

$$\sigma_s^* = \frac{\Upsilon_{\varphi^*,3,1} + \Upsilon_{\varphi^*,3,2} E - \Upsilon_{\varphi^*,3,3} C_s}{\varphi^*/y_\alpha - \Upsilon_{\varphi^*,3,3} y_\alpha}. \tag{C.19}$$

It follows from Eqs. (C.16) and (C.19) that:

$$E_s^* = y_\alpha \left(\frac{\Upsilon_{\varphi^*,3,1} + \Upsilon_{\varphi^*,3,2} E - \Upsilon_{\varphi^*,3,3} C_s}{\varphi^*/y_\alpha - \Upsilon_{\varphi^*,3,3} y_\alpha} \right) - C_s. \tag{C.20}$$

Using Eqs. (C.13) and (C.15), we have:

$$\boldsymbol{w}^* = (\boldsymbol{\Sigma}_{\varphi^*}^{-1} \boldsymbol{\Psi}) \boldsymbol{\Upsilon}_{\varphi^*} \boldsymbol{\kappa}_{E_s^*}. \tag{C.21}$$

It follows from Eq. (C.21) that:

$$(\sigma_s^*)^2 = [(\boldsymbol{\Sigma}_{\varphi^*}^{-1} \boldsymbol{\Psi}) \boldsymbol{\Upsilon}_{\varphi^*} \boldsymbol{\kappa}_{E_s^*}]' \boldsymbol{\Sigma}_s [(\boldsymbol{\Sigma}_{\varphi^*}^{-1} \boldsymbol{\Psi}) \boldsymbol{\Upsilon}_{\varphi^*} \boldsymbol{\kappa}_{E_s^*}]. \tag{C.22}$$

Using the right-hand sides of Eqs. (C.19) and (C.20) in, respectively, the left- and right-hand sides of Eq. (C.22), we obtain an equation with a single unknown, φ^*, which can be found numerically. ■

References (cited in Appendix C)

Beck, A., 2014. *Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB*. Society for Industrial and Applied Mathematics.

Horst, R., Pardalos, P. M., Thoai, N. V., 2000. *Introduction to Global Optimization*, Boston, Kluwer Academic Publishers.

[C.3] The fact that $\left(\varphi^*/y_\alpha - \Upsilon_{\varphi^*,3,3} y_\alpha\right) \neq 0$ follows from the more general claim that if $\alpha > \alpha_s$, $E \in \mathbb{R}$, and $\varphi \in (0,1)$, then $\Upsilon_{\varphi,3,3} > \varphi/y_\alpha^2$. Next, we provide a proof of this claim. Fix $\alpha > \alpha_s$, $E \in \mathbb{R}$, and $\varphi \in (0,1)$. Since $\alpha > \alpha_s$, we have $y_\alpha^2 > g_s$. Let σ_{s,E_s}^2 denote the stressed variance of the portfolio that solves $\min_{\boldsymbol{w} \in \mathbb{R}^N} \boldsymbol{w}' \boldsymbol{\Sigma}_s \boldsymbol{w}$ subject to $\boldsymbol{\Psi}' \boldsymbol{w} = \boldsymbol{\kappa}_{E_s}$. Alexander and Baptista (2017, Lemma 2) show that $\sigma_{s,E_s}^2 = h_s + \frac{(E_s - i_s)^2}{g_s}$ where $g_s \in \mathbb{R}_{++}$ and $h_s \in \mathbb{R}_{++}$ are defined earlier and $i_s \in \mathbb{R}$. Hence, $\lim_{E_s \to \infty} \frac{\sigma_{s,E_s}^2}{E_s^2} = \frac{1}{g_s}$. Let σ_{φ,E_s}^2 denote the weighted average of unstressed and stressed variances with unstressed and stressed variance weights of, respectively, $1-\varphi$ and φ of the portfolio that solves $\min_{\boldsymbol{w} \in \mathbb{R}^N} \boldsymbol{w}' \boldsymbol{\Sigma}_\varphi \boldsymbol{w}$ subject to $\boldsymbol{\Psi}' \boldsymbol{w} = \boldsymbol{\kappa}_{E_s}$. Let σ_{s,φ,E_s}^2 denote the stressed variance of this portfolio. Note that $\sigma_{\varphi,E_s}^2 = \boldsymbol{\kappa}_{E_s}' \boldsymbol{\Upsilon}_\varphi \boldsymbol{\kappa}_{E_s}$. Hence, $\lim_{E_s \to \infty} \frac{\sigma_{\varphi,E_s}^2}{E_s^2} = \Upsilon_{\varphi,3,3}$. It follows that $\Upsilon_{\varphi,3,3} = \lim_{E_s \to \infty} \frac{\sigma_{\varphi,E_s}^2}{E_s^2} \geq \varphi \left(\lim_{E_s \to \infty} \frac{\sigma_{s,\varphi,E_s}^2}{E_s^2} \right) \geq \varphi \left(\lim_{E_s \to \infty} \frac{\sigma_{s,E_s}^2}{E_s^2} \right) = \frac{\varphi}{g_s} > \frac{\varphi}{y_\alpha^2}$.

Online Appendix D: adding a risk-free asset

This Appendix extends our results to the case where a risk-free asset is present. The unstressed and stressed returns of the risk-free asset are, respectively, $r_f \in \mathbb{R}$ and $r_{f,s} \in \mathbb{R}$. We assume that $r_f \neq \frac{a}{c}$, $r_f \neq \frac{a_s}{c_s}$, and $r_{f,s} \neq \frac{d_s}{c_s}$ as well as $r_f \neq \frac{a_\varphi}{c_\varphi}$ and $r_{f,s} \neq \frac{d_\varphi}{c_\varphi}$ for any $\varphi \in (0,1)$.

A portfolio is now an $(N+1) \times 1$ vector $\overline{\boldsymbol{w}} = [\underline{\boldsymbol{w}}' \quad \overline{w}_{N+1}]'$ where $\underline{\boldsymbol{w}}$ and $\overline{w}_{N+1} = 1 - \underline{\boldsymbol{w}}'\mathbf{1}$ denote, respectively, the $N \times 1$ vector of risky asset weights and the risk-free asset weight.

D.1. The M-V frontier

Let $\mathbf{0}$ denote the $N \times 1$ zero vector. Merton (1972) uses portfolios $\overline{\boldsymbol{w}}_0 \equiv [\mathbf{0}' \quad 1]'$ and $\overline{\boldsymbol{w}}_1 \equiv \left[\frac{(\boldsymbol{\mu}-\mathbf{1}r_f)'\boldsymbol{\Sigma}^{-1}}{a-cr_f} \quad 0\right]'$ to characterize the M-V frontier. The portfolio on it with an expected return of $E \in \mathbb{R}$ is:

$$\overline{\boldsymbol{w}}_E \equiv \overline{\theta}_E \overline{\boldsymbol{w}}_0 + (1-\overline{\theta}_E)\overline{\boldsymbol{w}}_1 \tag{D.1}$$

where $\overline{\theta}_E \equiv \frac{E-[(b-ar_f)/(a-cr_f)]}{r_f-[(b-ar_f)/(a-cr_f)]}$.

D.2. The M-SCVaR frontier

Let $j_s \equiv c_s r_f^2 - 2a_s r_f + b_s$, $k_s \equiv c_s r_f r_{f,s} - a_s r_{f,s} - d_s r_f + f_s$, $l_s \equiv c_s r_{f,s}^2 - 2d_s r_{f,s} + e_s$, and $\overline{g}_s \equiv l_s - \frac{k_s^2}{j_s}$ denote constants with $j_s > 0$, $l_s > 0$, and $\overline{g}_s > 0$. Also, let $\overline{\alpha}_s$ be the constant for which $y_{\overline{\alpha}_s} = \sqrt{\overline{g}_s}$. Alexander and Baptista (2017) use portfolios $\overline{\boldsymbol{w}}_0$, $\overline{\boldsymbol{w}}_{1,s} \equiv \left[\frac{(\boldsymbol{\mu}-\mathbf{1}r_f)'\boldsymbol{\Sigma}_s^{-1}}{a_s-c_s r_f} \quad 0\right]'$, and $\overline{\boldsymbol{w}}_{2,s} \equiv \left[\frac{(\boldsymbol{\mu}_s-\mathbf{1}r_{f,s})'\boldsymbol{\Sigma}_s^{-1}}{d_s-c_s r_{f,s}} \quad 0\right]'$ to characterize the M-SCVaR frontier when $\alpha > \overline{\alpha}_s$. The portfolio on it with an expected return of $E \in \mathbb{R}$ is:

$$\overline{\boldsymbol{w}}_{\alpha,E} \equiv \overline{\theta}_{0,\alpha,E}\, \overline{\boldsymbol{w}}_0 + \overline{\theta}_{1,\alpha,E}\, \overline{\boldsymbol{w}}_{1,s} + \left(1 - \overline{\theta}_{0,\alpha,E} - \overline{\theta}_{1,\alpha,E}\right) \overline{\boldsymbol{w}}_{2,s} \tag{D.2}$$

where $\overline{\theta}_{0,\alpha,E} \in \mathbb{R}$ and $\overline{\theta}_{1,\alpha,E} \in \mathbb{R}$ are defined in Appendix C.

D.3. The SCVaR-constrained M-V frontier

For any $\varphi \in (0,1)$, $\overline{\boldsymbol{w}}_{1,\varphi} \equiv \left[\frac{(\boldsymbol{\mu}-\mathbf{1}r_f)'\boldsymbol{\Sigma}_\varphi^{-1}}{a_\varphi-c_\varphi r_f} \quad 0\right]'$ and $\overline{\boldsymbol{w}}_{2,\varphi} \equiv \left[\frac{(\boldsymbol{\mu}_s-\mathbf{1}r_{f,s})'\boldsymbol{\Sigma}_\varphi^{-1}}{d_\varphi-c_\varphi r_{f,s}} \quad 0\right]'$ are two portfolios. We next use $\overline{\boldsymbol{w}}_0$ and $\{(\overline{\boldsymbol{w}}_{1,\varphi}, \overline{\boldsymbol{w}}_{2,\varphi})\}_{\varphi\in(0,1)}$ to characterize the SCVaR-constrained M-V frontier.

Theorem 2. *Fix any confidence level $\alpha > \overline{\alpha}_s$, any bound $C_s \in \mathbb{R}$, and any expected return $E \in \mathbb{R}$. (i) If $C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$, then there is no portfolio on the SCVaR-constrained M-V frontier with confidence level α and bound C_s that has an expected return of E. (ii) If $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$, then such a portfolio is:*

$$\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\boldsymbol{w}}_{\alpha,E}. \tag{D.3}$$

(iii) If $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$, then it is:

$$\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\theta}_{0,\alpha,C_s,E}\, \overline{\boldsymbol{w}}_0 + \overline{\theta}_{1,\alpha,C_s,E}\, \overline{\boldsymbol{w}}_{1,\overline{\varphi}_{\alpha,C_s,E}} + (1 - \overline{\theta}_{0,\alpha,C_s,E} - \overline{\theta}_{1,\alpha,C_s,E})\, \overline{\boldsymbol{w}}_{2,\overline{\varphi}_{\alpha,C_s,E}} \tag{D.4}$$

where $\overline{\theta}_{0,\alpha,C_s,E} \in \mathbb{R}$, $\overline{\theta}_{1,\alpha,C_s,E} \in \mathbb{R}$, and $\overline{\varphi}_{\alpha,C_s,E} \in (0,1)$ are defined in Appendix C. (iv) If $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$, then it is:

$$\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\boldsymbol{w}}_E. \tag{D.5}$$

Theorem 2 is similar to Theorem 1 where a risk-free asset is absent. Hence, our discussion of the latter theorem extends to the former.

D.4. Example

We now add a risk-free asset to our numerical example. In doing so, we use US Treasury bills with a maturity range of 0–0.25 years (daily returns are obtained from Bloomberg).[D.1]

[D.1] The average return and stressed average return on US Treasury bills are, respectively, 0.02% and 0.04%.

Suppose that $C_s = 20\%$. Panel A of Fig. D2 plots various frontiers in M-SD space. Compared to panel A of Fig. 2 where a risk-free asset is absent, we note three minor differences. First, the portfolios on the unconstrained and SCVaR-constrained M-V frontiers with any given (feasible) expected return generally have slightly smaller SDs in panel A of Fig. D2.[D.2] Second, the range of expected returns where the SCVaR constraint does not bind widens slightly in panel A of Fig. D2. Third, the range of feasible expected returns with this constraint widens slightly in panel A of Fig. D2. Similar results hold if $C_s = 10\%$; compare panels A and B of Figs. D2. Since the effect of the presence of a risk-free asset on the location of the unconstrained and SCVaR-constrained M-V frontiers is modest, so is its effect on the results on the impact of an SCVaR constraint on various statistics of the optimal portfolio; compare panels A–D of Fig. D3 to panels A–D of Fig. 3.

Consider now the impact of an SCVaR constraint in the absence of the Volcker rule. The results in the presence of a risk-free asset shown in panels E–H of Fig. D3 are similar to those reported for panels E–H of Fig. 3.

D.5. Impact of the Volcker rule

When an SCVaR constraint is present, the results in panels A–D of Fig. D4 (with a risk-free asset) are similar to the results in panels A–D of Fig. 4 (without a risk-free asset). When an SCVaR constraint is absent, the results in panels E–H of Fig. D4 (with a risk-free asset) are similar to the results in panels E–H of Fig. 4 (without a risk-free asset).

D.6. Proofs of theoretical results

Proof of Theorem 2. Suppose that $\alpha > \overline{\alpha}_s$, $C_s \in \mathbb{R}$, and $E \in \mathbb{R}$. First, we show (i). Assume that $C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$. Since $\overline{\boldsymbol{w}}_{\alpha,E}$ is on the M-SCVaR frontier and $C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$, no portfolio with an expected return of E meets the SCVaR constraint. Hence, no portfolio exists on the SCVaR-constrained M-V frontier with an expected return of E. This completes the first part of our proof.

Second, we show (ii). Assume that $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$. Since $\overline{\boldsymbol{w}}_{\alpha,E}$ is on the M-SCVaR frontier, the only portfolio with an expected return of E that meets the SCVaR constraint is $\overline{\boldsymbol{w}}_{\alpha,E}$. Therefore, Eq. (D.3) holds. This completes the second part of our proof.

Third, we show (iii). Assume that $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. Note that $\overline{\boldsymbol{w}}^* \equiv \overline{\boldsymbol{w}}_{\alpha,C_s,E}$ solves:[D.3]

$$\min_{\boldsymbol{w}\in\mathbb{R}^N} \quad \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \tag{D.6}$$

$$s.t. \quad \boldsymbol{w}'(\boldsymbol{\mu} - \mathbf{1}r_f) = E - r_f \tag{D.7}$$

$$y_\alpha\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}_s\boldsymbol{w}} - \boldsymbol{w}'(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s}) \leq C_s + r_{f,s}. \tag{D.8}$$

A first-order condition for $\underline{\boldsymbol{w}}^*$ to solve problem (D.6) subject to constraints (D.7) and (D.8) is:

$$\boldsymbol{\Sigma}\underline{\boldsymbol{w}}^* + \lambda_4(\boldsymbol{\mu} - \mathbf{1}r_f) + \lambda_5\left[\frac{y_\alpha\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^*}{\sqrt{(\underline{\boldsymbol{w}}^*)'\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^*}} - (\boldsymbol{\mu}_s - \mathbf{1}r_{f,s})\right] = \mathbf{0} \tag{D.9}$$

where $\lambda_4 \in \mathbb{R}$ and $\lambda_5 \in \mathbb{R}_+$ are multipliers associated with these constraints.[D.4] Since $C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$, $\overline{\boldsymbol{w}}^*$ is not on the M-V frontier. Hence, $\lambda_5 > 0$. Let:

$$\overline{\eta} \equiv \lambda_5\left[\frac{y_\alpha}{\sqrt{(\underline{\boldsymbol{w}}^*)'\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^*}}\right]. \tag{D.10}$$

Since $y_\alpha > 0$, $\lambda_5 > 0$, and $(\underline{\boldsymbol{w}}^*)'\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^* > 0$, we have $\overline{\eta} > 0$. Eqs. (D.9) and (D.10) imply that:

$$(\boldsymbol{\Sigma} + \overline{\eta}\boldsymbol{\Sigma}_s)\,\underline{\boldsymbol{w}}^* = -\lambda_4(\boldsymbol{\mu} - \mathbf{1}r_f) + \lambda_5(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s}). \tag{D.11}$$

[D.2] For the expected return equal to that of the tangency portfolio, the corresponding portfolio on the M-V frontier when a risk-free asset is present coincides with the portfolio on such a frontier when this asset is absent.

[D.3] The existence and uniqueness of the solution to this problem follows from the arguments in footnote C.1.

[D.4] Note that the Kuhn-Tucker conditions are necessary and sufficient for $\underline{\boldsymbol{w}}^*$ to solve this problem.

Let:

$$\overline{\varphi}^* \equiv \frac{\overline{\eta}}{1+\overline{\eta}}. \tag{D.12}$$

Since $\overline{\eta} > 0$, Eq. (D.12) implies that $\overline{\varphi}^* \in (0,1)$. Dividing both sides of Eq. (D.11) by $(1+\overline{\eta})$ and using Eq. (D.12), we have:

$$\mathbf{\Sigma}_{\overline{\varphi}^*} \underline{\boldsymbol{w}}^* = -\left(\frac{\lambda_4}{1+\overline{\eta}}\right)(\boldsymbol{\mu} - \mathbf{1}r_f) + \left(\frac{\lambda_5}{1+\overline{\eta}}\right)(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s}). \tag{D.13}$$

Premultiplying Eq. (D.13) by $\mathbf{\Sigma}_{\overline{\varphi}^*}^{-1}$, we obtain:

$$\underline{\boldsymbol{w}}^* = -\left(\frac{\lambda_4}{1+\overline{\eta}}\right)[\mathbf{\Sigma}_{\overline{\varphi}^*}^{-1}(\boldsymbol{\mu} - \mathbf{1}r_f)] + \left(\frac{\lambda_5}{1+\overline{\eta}}\right)[\mathbf{\Sigma}_{\overline{\varphi}^*}^{-1}(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s})]. \tag{D.14}$$

Let:

$$\overline{\theta}_{0,\alpha,C_s,E} \equiv 1 + \left(\frac{\lambda_4}{1+\overline{\eta}}\right)(a_{\overline{\varphi}^*} - c_{\overline{\varphi}^*} r_f) - \left(\frac{\lambda_5}{1+\overline{\eta}}\right)(d_{\overline{\varphi}^*} - c_{\overline{\varphi}^*} r_{f,s}) \tag{D.15}$$

and:

$$\overline{\theta}_{1,\alpha,C_s,E} \equiv -\left(\frac{\lambda_4}{1+\overline{\eta}}\right)(a_{\overline{\varphi}^*} - c_{\overline{\varphi}^*} r_f). \tag{D.16}$$

Using Eqs. (D.15) and (D.16), we have:

$$1 - \overline{\theta}_{0,\alpha,C_s,E} - \overline{\theta}_{1,\alpha,C_s,E} = \left(\frac{\lambda_5}{1+\overline{\eta}}\right)(d_{\overline{\varphi}^*} - c_{\overline{\varphi}^*} r_{f,s}). \tag{D.17}$$

Eqs. (D.14)–(D.17) imply that Eq. (D.4) holds with $\overline{\varphi}_{\alpha,C_s,E} = \overline{\varphi}^*$. This completes the third part of our proof.

Fourth, we show (iv). Assume that $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. Since $\overline{\boldsymbol{w}}_E$ is on the M-V frontier and meets the SCVaR constraint, Eq. (D.5) holds. This completes the fourth part of our proof. ■

An approach to find $\overline{\varphi}_{\alpha,C_s,E}$. Suppose that $\alpha > \overline{\alpha}_s$, $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$, and $E \in \mathbb{R}$. The following notation is useful. Let $\overline{\mathbf{\Psi}} \equiv [\boldsymbol{\mu} - \mathbf{1}r_f \quad \boldsymbol{\mu}_s - \mathbf{1}r_{f,s}]$ denote a $N \times 2$ matrix. For any $E_s \in \mathbb{R}$, let $\overline{\boldsymbol{\kappa}}_{E_s} \equiv [E - r_f \quad E_s - r_{f,s}]'$ denote a 2×1 vector. For any $\varphi \in (0,1)$, let $\overline{\mathbf{\Upsilon}}_\varphi \equiv (\overline{\mathbf{\Psi}}' \mathbf{\Sigma}_\varphi^{-1} \overline{\mathbf{\Psi}})^{-1}$ denote a 2×2 matrix. For any $(m_1, m_2) \in \{1,2,3\} \times \{1,2,3\}$, let $\overline{\Upsilon}_{\varphi,m_1,m_2}$ denote the element in row m_1 and column m_2 of $\overline{\mathbf{\Upsilon}}_\varphi$. Let $\overline{E}_s^* \equiv E_s[r_{\overline{\boldsymbol{w}}^*}]$ and $\overline{\sigma}_s^* \equiv \sigma_s[r_{\overline{\boldsymbol{w}}^*}]$ where $\overline{\boldsymbol{w}}^*$ is defined in the proof of Theorem 2. Also, let $\overline{\boldsymbol{\pi}}^* \equiv [\overline{\pi}_1^* \quad \overline{\pi}_2^*]'$ where $\overline{\pi}_1^* \equiv -\frac{\lambda_4}{1+\overline{\eta}}$ and $\overline{\pi}_2^* \equiv \frac{\lambda_5}{1+\overline{\eta}}$. Here, λ_4 and λ_5, and $\overline{\eta}$ are defined in the proof of Theorem 2.

Using Eq. (D.14), we have:

$$\underline{\boldsymbol{w}}^* = (\mathbf{\Sigma}_{\overline{\varphi}^*}^{-1} \overline{\mathbf{\Psi}}) \overline{\boldsymbol{\pi}}^*. \tag{D.18}$$

Premultiplying Eq. (D.18) with $\overline{\mathbf{\Psi}}'$, we obtain:

$$\overline{\boldsymbol{\kappa}}_{\overline{E}_s^*} = (\overline{\mathbf{\Psi}}' \mathbf{\Sigma}_{\overline{\varphi}^*}^{-1} \overline{\mathbf{\Psi}}) \overline{\boldsymbol{\pi}}^*. \tag{D.19}$$

Premultiplying Eq. (D.19) by $\overline{\mathbf{\Upsilon}}_{\overline{\varphi}^*} = (\overline{\mathbf{\Psi}}' \mathbf{\Sigma}_{\overline{\varphi}^*}^{-1} \overline{\mathbf{\Psi}})^{-1}$, we have:

$$\overline{\boldsymbol{\pi}}^* = \overline{\mathbf{\Upsilon}}_{\overline{\varphi}^*} \overline{\boldsymbol{\kappa}}_{\overline{E}_s^*}. \tag{D.20}$$

Since the SCVaR constraint binds:

$$\overline{E}_s^* = y_\alpha \overline{\sigma}_s^* - C_s. \tag{D.21}$$

It follows from Eqs. (D.20) and (D.21) as well as the definition of $\overline{\boldsymbol{\kappa}}_{\overline{E}_s^*}$ that:

$$\overline{\pi}_2^* = \overline{\Upsilon}_{\overline{\varphi}^*,2,1}(E - r_f) + \overline{\Upsilon}_{\overline{\varphi}^*,2,2}(y_\alpha \overline{\sigma}_s^* - C_s - r_{f,s}). \tag{D.22}$$

The definition of $\overline{\pi}_2^*$ as well as Eqs. (D.10) and (D.12) imply that:

$$\overline{\pi}_2^* = (\overline{\varphi}^*/y_\alpha)\overline{\sigma}_s^*. \tag{D.23}$$

Noting that $\left(\overline{\varphi}^*/y_\alpha - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}y_\alpha\right) \neq 0$, Eqs. (D.22) and (D.23) imply that:[D.5]

$$\overline{\sigma}_s^* = \frac{\overline{\Upsilon}_{\overline{\varphi}^*,2,1}\,(E - r_f) - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}(C_s + r_{f,s})}{\overline{\varphi}^*/y_\alpha - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}y_\alpha}. \tag{D.24}$$

It follows from Eqs. (D.21) and (D.24) that:

$$\overline{E}_s^* = y_\alpha \left[\frac{\overline{\Upsilon}_{\overline{\varphi}^*,2,1}\,(E - r_f) - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}(C_s + r_{f,s})}{\overline{\varphi}^*/y_\alpha - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}y_\alpha}\right] - C_s. \tag{D.25}$$

Using Eqs. (D.18) and (D.20), we have:

$$\underline{\boldsymbol{w}}^* = (\boldsymbol{\Sigma}_{\overline{\varphi}^*}^{-1}\overline{\boldsymbol{\Psi}})\overline{\boldsymbol{\Upsilon}}_{\overline{\varphi}^*}\overline{\boldsymbol{\kappa}}_{\overline{E}_s^*}. \tag{D.26}$$

It follows from Eq. (D.26) that:

$$(\overline{\sigma}_s^*)^2 = [(\boldsymbol{\Sigma}_{\overline{\varphi}^*}^{-1}\overline{\boldsymbol{\Psi}})\overline{\boldsymbol{\Upsilon}}_{\overline{\boldsymbol{\varphi}}^*}\overline{\boldsymbol{\kappa}}_{\overline{E}_s^*}]'\boldsymbol{\Sigma}_s[(\boldsymbol{\Sigma}_{\overline{\varphi}^*}^{-1}\overline{\boldsymbol{\Psi}})\overline{\boldsymbol{\Upsilon}}_{\overline{\boldsymbol{\varphi}}^*}\overline{\boldsymbol{\kappa}}_{\overline{E}_s^*}]. \tag{D.27}$$

Using the right-hand sides of Eqs. (D.24) and (D.25) in, respectively, the left- and right-hand sides of Eq. (D.27), we obtain an equation with a single unknown, $\overline{\varphi}^*$, which can be found numerically. ■

[D.5] The fact that $\left(\overline{\varphi}^*/y_\alpha - \overline{\Upsilon}_{\overline{\varphi}^*,2,2}y_\alpha\right) \neq 0$ follows from the more general claim that if $\alpha > \overline{\alpha}_s$, $E \in \mathbb{R}$, and $\varphi \in (0,1)$, then $\overline{\Upsilon}_{\varphi,2,2} > \varphi/y_\alpha^2$. A proof of this claim appears next. Fix $\alpha > \overline{\alpha}_s$, $E \in \mathbb{R}$, and $\varphi \in (0,1)$. Since $\alpha > \overline{\alpha}_s$, we have $y_\alpha^2 > \overline{g}_s$. Let $\overline{\sigma}_{s,E_s}^2$ denote the stressed variance of the portfolio that solves $\min_{\boldsymbol{w}\in\mathbb{R}^N} \boldsymbol{w}'\boldsymbol{\Sigma}_s\boldsymbol{w}$ subject to $\overline{\boldsymbol{\Psi}}'\boldsymbol{w}=\overline{\boldsymbol{\kappa}}_{E_s}$. Alexander and Baptista (2017, Lemma 4) show that $\overline{\sigma}_{s,E_s}^2 = \overline{h}_s + \frac{(E_s-\overline{i}_s)^2}{\overline{g}_s}$ where $\overline{g}_s \in \mathbb{R}_{++}$ is defined earlier, $\overline{h}_s \in \mathbb{R}_+$, and $\overline{i}_s \in \mathbb{R}$. Hence, $\lim_{E_s\to\infty} \frac{\overline{\sigma}_{s,E_s}^2}{E_s^2} = \frac{1}{\overline{g}_s}$. Let $\overline{\sigma}_{\varphi,E_s}^2$ denote the weighted average of unstressed and stressed variances with unstressed and stressed variance weights of, respectively, $1-\varphi$ and φ of the portfolio that solves $\min_{\boldsymbol{w}\in\mathbb{R}^N} \boldsymbol{w}'\boldsymbol{\Sigma}_\varphi\boldsymbol{w}$ subject to $\overline{\boldsymbol{\Psi}}'\boldsymbol{w}=\overline{\boldsymbol{\kappa}}_{E_s}$. Let $\overline{\sigma}_{s,\varphi,E_s}^2$ denote the stressed variance of this portfolio. Note that $\overline{\sigma}_{\varphi,E_s}^2 = \overline{\boldsymbol{\kappa}}_{E_s}'\overline{\boldsymbol{\Upsilon}}_\varphi\overline{\boldsymbol{\kappa}}_{E_s}$. Hence, $\lim_{E_s\to\infty}\frac{\overline{\sigma}_{\varphi,E_s}^2}{E_s^2} = \overline{\Upsilon}_{\varphi,2,2}$. It follows that $\overline{\Upsilon}_{\varphi,2,2} = \lim_{E_s\to\infty}\frac{\overline{\sigma}_{\varphi,E_s}^2}{E_s^2} \geq \varphi\left(\lim_{E_s\to\infty}\frac{\overline{\sigma}_{s,\varphi,E_s}^2}{E_s^2}\right) \geq \varphi\left(\lim_{E_s\to\infty}\frac{\overline{\sigma}_{s,E_s}^2}{E_s^2}\right) = \frac{\varphi}{\overline{g}_s} > \frac{\varphi}{y_\alpha^2}$.

Fig. D1. The M-V, M-SCVaR, and SCVaR-constrained M-V frontiers when a risk-free asset is present

Suppose that a risk-free asset is present. The solid and dotted lines show, respectively, the M-V and M-SCVaR frontiers. The bottom and top dashed curves along with the portion of the solid curve between points J_6 and J_7 show the SCVaR-constrained M-V frontier. Let E_5–E_8 denote the expected returns associated with, respectively, points J_5–J_8. For any expected return either less than E_5 or more than E_8, a portfolio on this frontier does not exist. For the expected return of either E_5 or E_8, such a portfolio is on the M-SCVaR frontier. For any expected return strictly between either E_5 and E_6 or E_7 and E_8, the portfolio is strictly between the M-V and M-SCVaR frontiers. For any expected return between (and including) E_7 and E_8, the portfolio is on the M-V frontier.

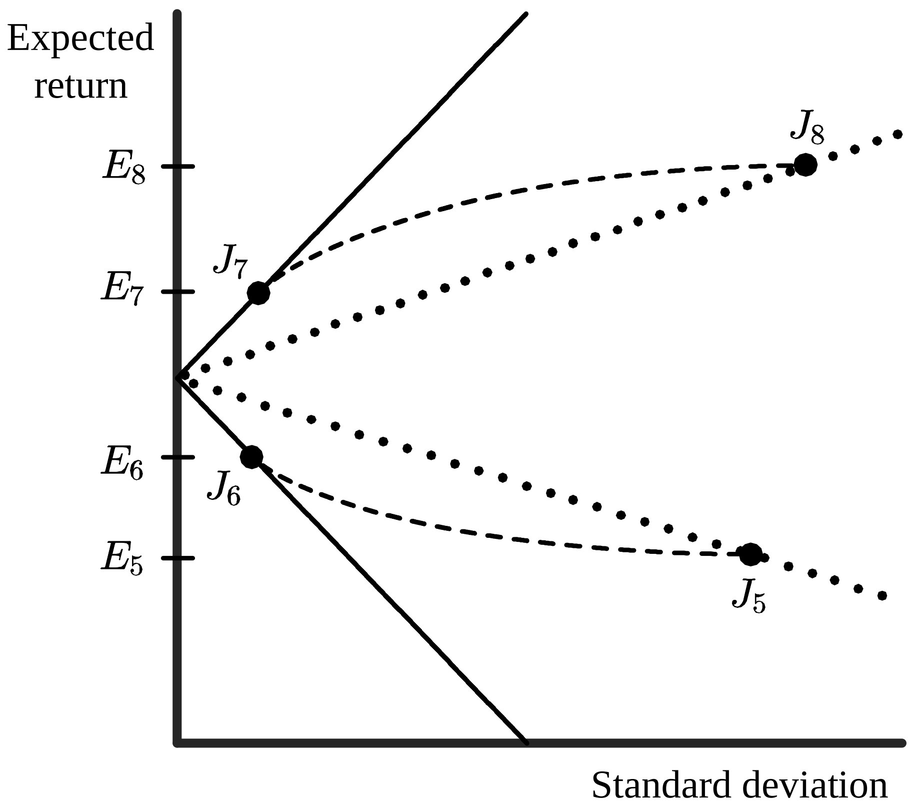


Fig. D2. The size of the bound and the location of the SCVaR-constrained M-V frontier relative to that of the M-V and M-SCVaR frontiers when a risk-free asset is present

Panels A and B plot various frontiers in M-SD space when a risk-free asset is present. In both panels, the solid curve shows the M-V frontier, whereas the dotted curve shows the M-SCVaR frontier with a confidence level, α, of 97.5%. In panel A, the bottom and top dashed curves along with the portion of the solid curve between points L_{10} and L_{11} show the SCVaR-constrained M-V frontier with a confidence level of 97.5% and a bound, C_s, of 20%. For any expected return either (a) less than that associated with point L_9 or (b) more than that associated with point L_{12}, a portfolio on this frontier does not exist. For the expected return equal to that associated with either of these points, such a portfolio is on the M-SCVaR frontier. For any expected return either (a) strictly between those associated with points L_9 and L_{10}, or (b) strictly between those associated with points L_{11} and L_{12}, the portfolio is strictly between the M-V and M-SCVaR frontiers. For any expected return between (and including) those associated with points L_{10} and L_{11}, the portfolio is on the M-V frontier. In panel B, the bound is 10% and points L_{13}–L_{16} represent the right- and left-end of the dashed curves.

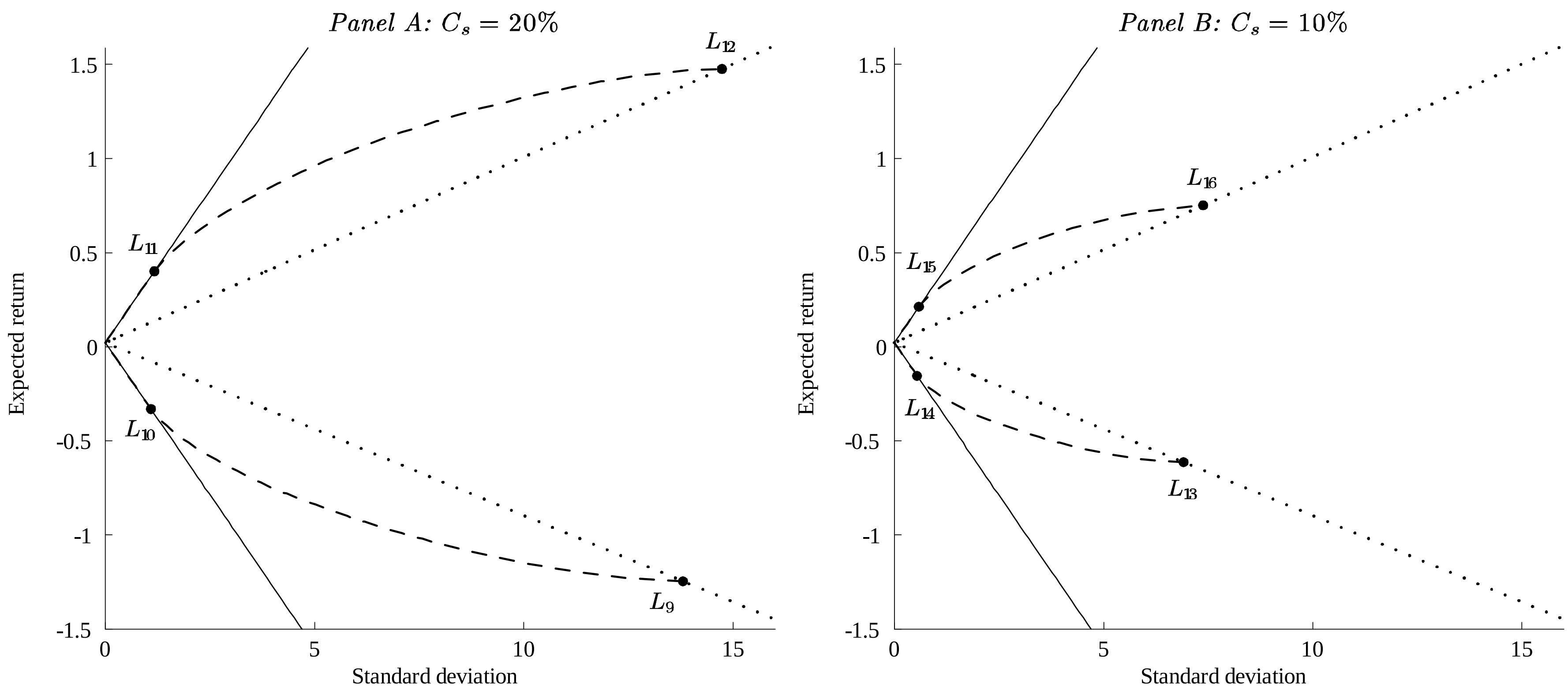


Fig. D3. Relative change in various statistics of the optimal portfolio due to an SCVaR constraint when a risk-free asset is present

Fix an M-V objective function with risk aversion coefficient γ as well as an SCVaR constraint with confidence level α and bound C_s. Fig. D3 examines the relative change in the SD, SCVaR, SD-to-SCVaR ratio, and CER of the optimal portfolio due to this SCVaR constraint when a risk-free asset is present. Panels A and E consider SD. Panels B and F consider SCVaR. Panels C and G consider the SD-to-SCVaR ratio. Panels D and H consider CER. In each panel, the thick and thin dashed lines show the relative change as a function of $\gamma \in [0.1, 10]$ when the confidence level is α is 97.5% and the bound is, respectively, 10% and 20%. While the Volcker rule is present in panels A–D, it is absent in panels E–H. SCVaR is given by: (i) Eq. (4) if the Volcker rule is present; and (ii) Eq. (13) if it is absent. Relative changes are reported in percentage points.

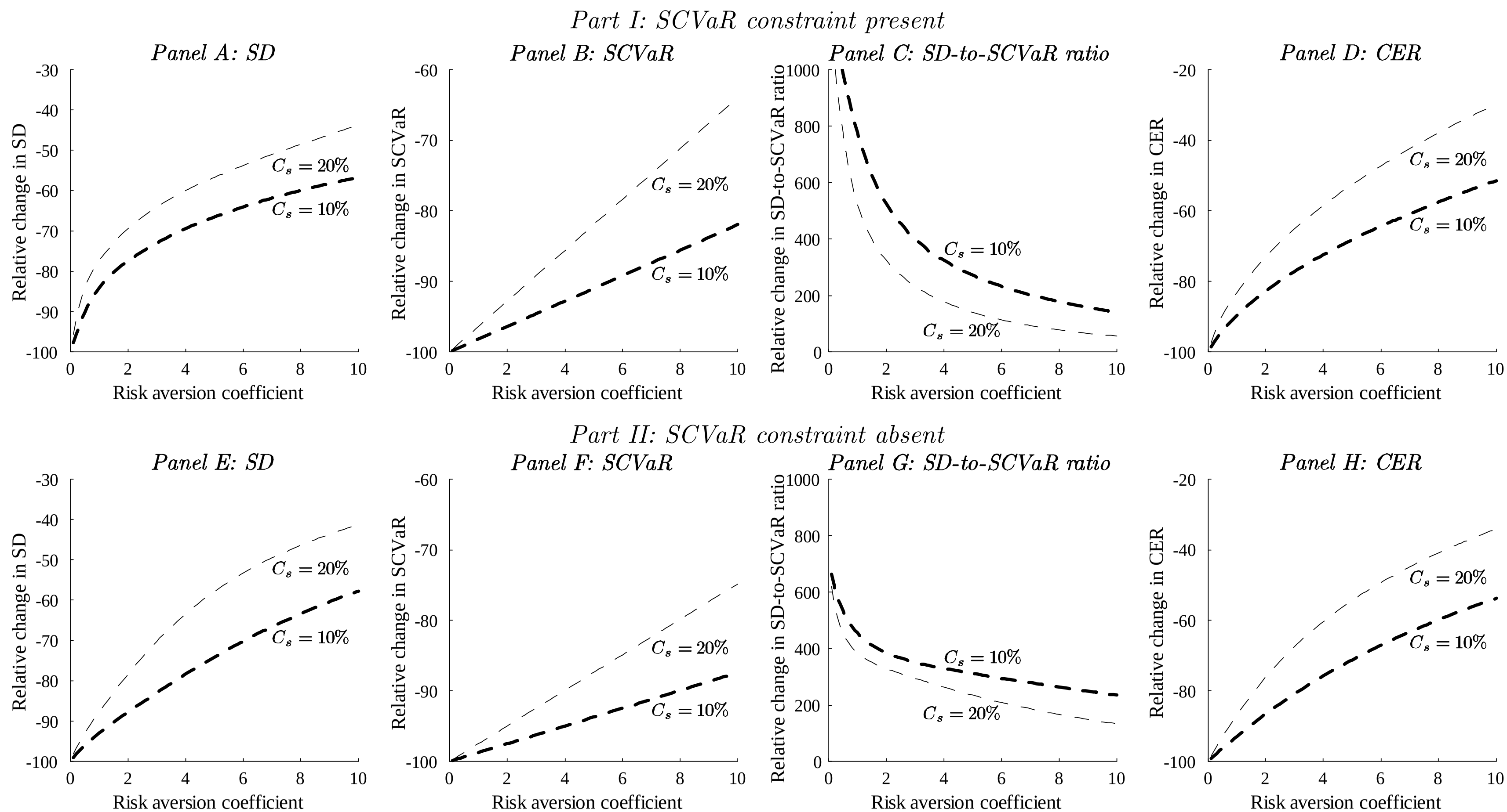
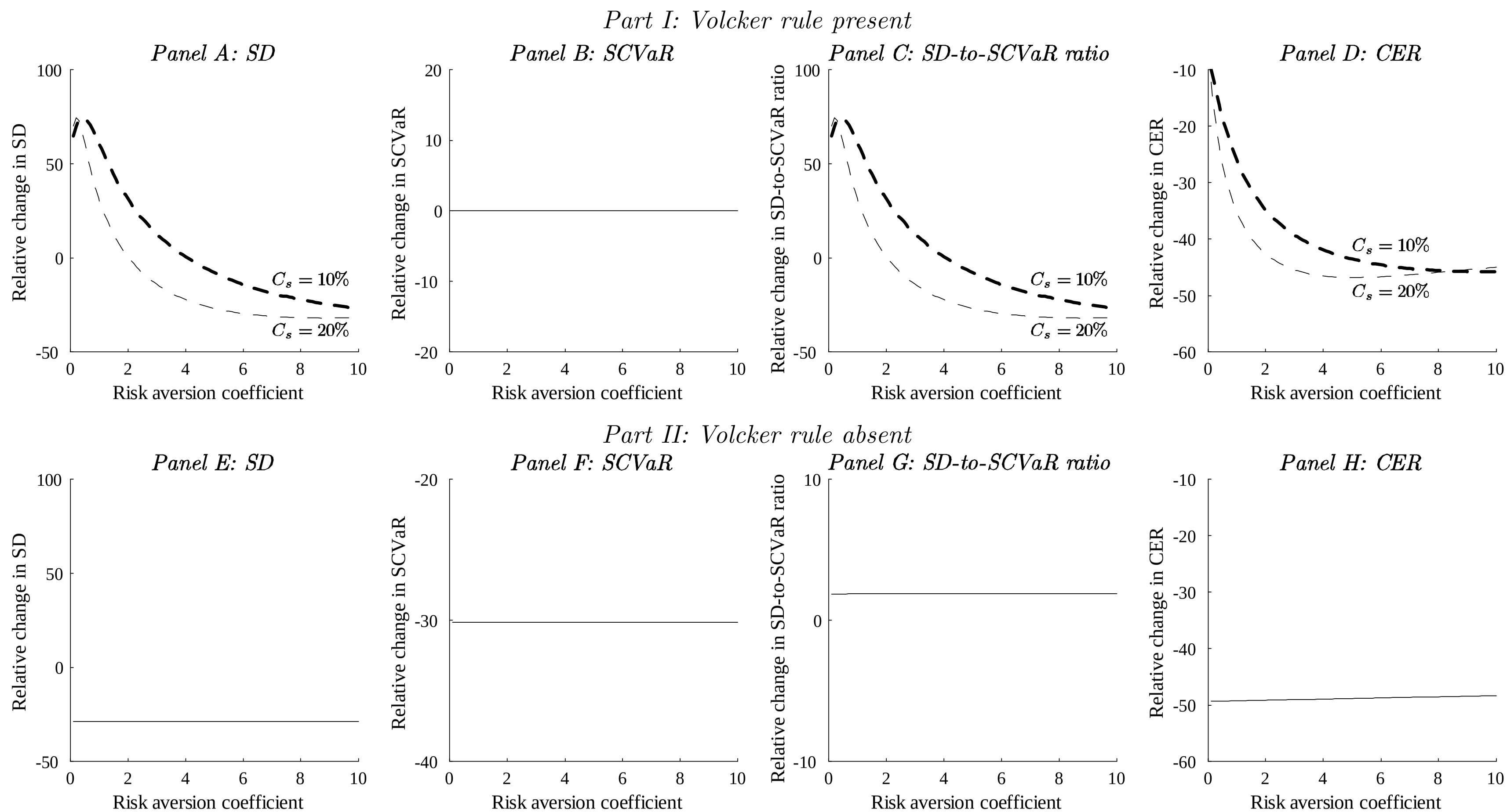


Fig. D4. Relative change in various statistics of the optimal portfolio due to the Volcker rule when a risk-free asset is present

Fix an M-V objective function with risk aversion coefficient γ as well as an SCVaR constraint with confidence level α and bound C_s. Fig. 4 examines the relative change in the SD, SCVaR, SD-to-SCVaR ratio, and CER of the optimal portfolio due to the Volcker rule when a risk-free asset is present. Panels A and E consider SD. Panels B and F consider SCVaR. Panels C and G consider the SD-to-SCVaR ratio. Panels D and H consider CER. In each panel, the thick and thin dashed lines show the relative change as a function of $\gamma \in [0.1, 10]$ when the confidence level is α is 97.5% and the bound is, respectively, 10% and 20%. While an SCVaR constraint is present in panels A–D, it is absent in panels E–H. SCVaR is given by: (i) Eq. (4) if the Volcker rule is present; and (ii) Eq. (18) if it is absent. Relative changes are reported in percentage points.



Online Appendix E: two properties of portfolios on the SCVaR-constrained M-V frontier

This Appendix presents two properties of portfolios on the SCVaR-constrained M-V frontier.

E.1. Absence of a risk-free asset

The following two corollaries of Theorem 1 consider the case where a risk-free asset is absent.

Corollary 1. *Fix any confidence level* $\alpha > \alpha_s$ *and any bound* $C_s \geq C_{s,\alpha,\boldsymbol{w}_{E,\alpha}}$ *where* $E \in \mathbb{R}$ *is an expected return. Portfolio* $\boldsymbol{w}_{\alpha,C_s,E}$ *minimizes a weighted average of unstressed and stressed variances among the portfolios with the same unstressed and stressed expected returns. The corresponding unstressed and stressed variance weights sum to 100% and the stressed variance weight is: (i) 100% if* $C_s = C_{s,\alpha,\boldsymbol{w}_{E,\alpha}}$*; (ii)* $\varphi_{\alpha,C_s,E}$ *if* $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C_s < C_{s,\alpha,\boldsymbol{w}_E}$*; and (iii) 0% if* $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$*.*

Proof. Suppose that $\alpha > \alpha_s$ and $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$ where $E \in \mathbb{R}$. Let $E_{s,\alpha,C_s,E}$ denote the stressed expected return of $\boldsymbol{w}_{\alpha,C_s,E}$. First, assume that $C_s = C_{s,\alpha,\boldsymbol{w}_{\alpha,E}}$. Eq. (8) says that $\boldsymbol{w}_{\alpha,C_s,E} = \boldsymbol{w}_{\alpha,E}$. Hence, $\boldsymbol{w}_{\alpha,C_s,E}$ minimizes stressed variance among the portfolios with an expected return of E and a stressed expected return of $E_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is 100%.

Second, assume that $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C_s < C_{s,\alpha,\boldsymbol{w}_E}$. Eq. (C.21) implies that $\boldsymbol{w}_{\alpha,C_s,E} = \boldsymbol{w}^*$ minimizes the weighted average of unstressed and stressed variances with unstressed and stressed variances weights of, respectively, $1-\varphi^*$ and φ^* among the portfolios with an expected return of E and a stressed expected return of $E_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is $\varphi_{\alpha,C_s,E} = \varphi^*$.

Third, assume that $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$. Eq. (10) says that $\boldsymbol{w}_{\alpha,C_s,E} = \boldsymbol{w}_E$. Hence, $\boldsymbol{w}_{\alpha,C_s,E}$ minimizes variance among the portfolios with an expected return of E and a stressed expected return of $E_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is 0%. ■

Corollary 2. *Fix any confidence level* $\alpha > \alpha_s$ *and any bound* $C_s \geq C_{s,\alpha,\boldsymbol{w}_{E,\alpha}}$ *where* $E \in \mathbb{R}$ *is an expected return. Portfolio* $\boldsymbol{w}_{\alpha,C_s,E}$ *minimizes a weighted average of variance and SCVaR at confidence level* α *among the portfolios with the same expected return. The corresponding variance and SCVaR weights sum to 100% and the SCVaR weight is: (i) 100% if* $C_s = C_{s,\alpha,\boldsymbol{w}_{E,\alpha}}$*; (ii) strictly between 0% and 100% if* $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C_s < C_{s,\alpha,\boldsymbol{w}_E}$*; and (iii) 0% if* $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$*.*

Proof. Suppose that $\alpha > \alpha_s$ and $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$ where $E \in \mathbb{R}$. First, assume that $C_s = C_{s,\alpha,\boldsymbol{w}_{\alpha,E}}$. It follows from Eq. (8) that the SCVaR weight is 100%.

Second, assume that $C_{s,\alpha,\boldsymbol{w}_{\alpha,E}} < C_s < C_{s,\alpha,\boldsymbol{w}_E}$. Let $\zeta \equiv \frac{\lambda_3}{1/2+\lambda_3}$ where $\lambda_3 > 0$ is defined in the proof of Theorem 1. Since $\lambda_3 > 0$, we have $\zeta \in (0,1)$. Consider the problem:

$$\begin{aligned} \min_{\boldsymbol{w}\in\mathbb{R}^N} \quad & (1-\zeta)(\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}) + (\zeta)(y_\alpha\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}_s\boldsymbol{w}} - \boldsymbol{w}'\boldsymbol{\mu}_s) && \text{(E.1)}\\ s.t. \quad & \boldsymbol{w}'\boldsymbol{1} = 1 && \text{(E.2)}\\ & \boldsymbol{w}'\boldsymbol{\mu} = E. && \text{(E.3)} \end{aligned}$$

Sufficient conditions for $\boldsymbol{w}^*$ to solve problem (E.1) subject to constraints (E.2) and (E.3) are:

$$(\boldsymbol{w}^*)'\boldsymbol{1} = 1, \tag{E.4}$$

$$(\boldsymbol{w}^*)'\boldsymbol{\mu} = E, \tag{E.5}$$

and:

$$2(1-\zeta)\boldsymbol{\Sigma}\boldsymbol{w}^* + \zeta\left[\frac{y_\alpha\boldsymbol{\Sigma}_s\boldsymbol{w}^*}{\sqrt{(\boldsymbol{w}^*)'\boldsymbol{\Sigma}_s\boldsymbol{w}^*}} - \boldsymbol{\mu}_s\right] + \delta_1\boldsymbol{1} + \delta_2\boldsymbol{\mu} = \boldsymbol{0} \tag{E.6}$$

for some $(\delta_1,\delta_2) \in \mathbb{R}^2$. Since $\boldsymbol{w}^*$ is on the SCVaR-constrained M-V frontier, Eqs. (E.4) and (E.5) hold. Multiplying Eq. (C.5) by $\frac{1}{1/2+\lambda_3}$, we obtain:

$$\left(\frac{1}{1/2+\lambda_3}\right)(\boldsymbol{\Sigma}\boldsymbol{w}^* + \lambda_1\boldsymbol{1} + \lambda_2\boldsymbol{\mu}) + \left(\frac{\lambda_3}{1/2+\lambda_3}\right)\left[\frac{y_\alpha\boldsymbol{\Sigma}_s\boldsymbol{w}^*}{\sqrt{(\boldsymbol{w}^*)'\boldsymbol{\Sigma}_s\boldsymbol{w}^*}} - \boldsymbol{\mu}_s\right] = \boldsymbol{0}. \tag{E.7}$$

Using the definition of ζ and Eq. (E.7), we have:

$$2\left(1-\zeta\right)\left(\boldsymbol{\Sigma}\boldsymbol{w}^{*}+\lambda_{1}\mathbf{1}+\lambda_{2}\boldsymbol{\mu}\right)+\zeta\left[\frac{y_{\alpha}\boldsymbol{\Sigma}_{s}\boldsymbol{w}^{*}}{\sqrt{(\boldsymbol{w}^{*})'\boldsymbol{\Sigma}_{s}\boldsymbol{w}^{*}}}-\boldsymbol{\mu}_{s}\right]=\mathbf{0}. \tag{E.8}$$

It follows from Eq. (E.8) that Eq. (E.6) holds with $\delta_1 = 2\left(1-\zeta\right)\lambda_1$ and $\delta_2 = 2\left(1-\zeta\right)\lambda_2$.

Third, assume that $C_s \geq C_{s,\alpha,\boldsymbol{w}_E}$. It follows from Eq. (10) that the SCVaR weight is 0%. ■

E.2. Presence of a risk-free asset

The following two corollaries of Theorem 2 consider the case where a risk-free asset is present.

Corollary 3. *Fix any confidence level $\alpha > \overline{\alpha}_s$ and any bound $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$ where $E \in \mathbb{R}$ is an expected return. Portfolio $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$ minimizes a weighted average of unstressed and stressed variances among the portfolios with the same unstressed and stressed expected returns. The corresponding unstressed and stressed variance weights sum to 100% and the stressed variance weight is: (i) 100% if $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$; (ii) $\overline{\varphi}_{\alpha,C_s,E}$ if $C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$; and (iii) 0% if $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$.*

Proof. Suppose that $\alpha > \overline{\alpha}_s$ and $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$ where $E \in \mathbb{R}$. Let $\overline{E}_{s,\alpha,C_s,E}$ denote the stressed expected return of $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$. First, assume that $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$. Eq. (D.3) says that $\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\boldsymbol{w}}_{\alpha,E}$. Hence, $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$ minimizes stressed variance among the portfolios with an expected return of E and a stressed expected return of $\overline{E}_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is 100%.

Second, assume that $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. Eq. (D.26) implies that $\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\boldsymbol{w}}^*$ minimizes the weighted average of unstressed and stressed variances with unstressed and stressed variance weights of, respectively, $1-\overline{\varphi}^*$ and $\overline{\varphi}^*$ among the portfolios with an expected return of E and a stressed expected return of $\overline{E}_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is $\overline{\varphi}_{\alpha,C_s,E} = \overline{\varphi}^*$.

Third, assume that $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. Eq. (D.5) says that $\overline{\boldsymbol{w}}_{\alpha,C_s,E} = \overline{\boldsymbol{w}}_E$. Hence, $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$ minimizes variance among the portfolios with an expected return of E and a stressed expected return of $\overline{E}_{s,\alpha,C_s,E}$. It follows that the stressed variance weight is 0%. ■

Corollary 4. *Fix any confidence level $\alpha > \overline{\alpha}_s$ and any bound $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$ where $E \in \mathbb{R}$ is an expected return. Portfolio $\overline{\boldsymbol{w}}_{\alpha,C_s,E}$ minimizes a weighted average of variance and SCVaR at confidence level α among the portfolios with the same expected return. The corresponding variance and SCVaR weights sum to 100% and the SCVaR weight is: (i) 100% if $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$; (ii) strictly between 0% and 100% if $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$; and (iii) 0% if $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$.*

Proof. Suppose that $\alpha > \overline{\alpha}_s$ and $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_{E,\alpha}}$ where $E \in \mathbb{R}$. First, assume that $C_s = C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}}$. It follows from Eq. (D.3) that the SCVaR weight is 100%.

Second, assume that $C_{s,\alpha,\overline{\boldsymbol{w}}_{\alpha,E}} < C_s < C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. Let $\overline{\zeta} \equiv \frac{\lambda_5}{1/2+\lambda_5}$ where $\lambda_5 > 0$ is defined in the proof of Theorem 2. Since $\lambda_5 > 0$, we have $\overline{\zeta} \in (0,1)$. Consider the problem:

$$\min_{\boldsymbol{w}\in\mathbb{R}^N} \quad (1-\overline{\zeta})(\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w})+(\overline{\zeta})[y_{\alpha}\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}_s\boldsymbol{w}}-\boldsymbol{w}'\left(\boldsymbol{\mu}_s-\mathbf{1}r_{f,s}\right)-r_{f,s}] \tag{E.9}$$

$$s.t. \qquad \boldsymbol{w}'\left(\boldsymbol{\mu}-\mathbf{1}r_f\right)=E-r_f. \tag{E.10}$$

Sufficient conditions for $\underline{\boldsymbol{w}}^*$ to solve problem (E.9) subject to constraint (E.10) are:

$$(\underline{\boldsymbol{w}}^*)'\left(\boldsymbol{\mu}-\mathbf{1}r_f\right)=E-r_f \tag{E.11}$$

and:

$$2(1-\overline{\zeta})\boldsymbol{\Sigma}\underline{\boldsymbol{w}}^*+\overline{\zeta}\left[\frac{y_{\alpha}\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^*}{\sqrt{(\underline{\boldsymbol{w}}^*)'\boldsymbol{\Sigma}_s\underline{\boldsymbol{w}}^*}}-\left(\boldsymbol{\mu}_s-\mathbf{1}r_{f,s}\right)\right]+\delta_3\left(\boldsymbol{\mu}-\mathbf{1}r_f\right)=\mathbf{0} \tag{E.12}$$

for some $\delta_3 \in \mathbb{R}$. Since $\overline{\boldsymbol{w}}^*$ is on the SCVaR-constrained M-V frontier, Eq. (E.11) holds. Multiplying Eq. (D.9) by $\frac{1}{1/2+\lambda_5}$, we obtain:

$$\left(\frac{1}{1/2+\lambda_5}\right)\left[\boldsymbol{\Sigma}\underline{\boldsymbol{w}}^* + \lambda_4\left(\boldsymbol{\mu} - \mathbf{1}r_f\right)\right] + \left(\frac{\lambda_5}{1/2+\lambda_5}\right)\left[\frac{y_\alpha \boldsymbol{\Sigma}_s \underline{\boldsymbol{w}}^*}{\sqrt{(\underline{\boldsymbol{w}}^*)' \boldsymbol{\Sigma}_s \underline{\boldsymbol{w}}^*}} - \left(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s}\right)\right] = \mathbf{0}. \quad \text{(E.13)}$$

Using the definition of $\overline{\zeta}$ and Eq. (E.13), we have:

$$2\left(1-\overline{\zeta}\right)\left[\boldsymbol{\Sigma}\underline{\boldsymbol{w}}^* + \lambda_4\left(\boldsymbol{\mu} - \mathbf{1}r_f\right)\right] + \overline{\zeta}\left[\frac{y_\alpha \boldsymbol{\Sigma}_s \underline{\boldsymbol{w}}^*}{\sqrt{(\underline{\boldsymbol{w}}^*)' \boldsymbol{\Sigma}_s \underline{\boldsymbol{w}}^*}} - \left(\boldsymbol{\mu}_s - \mathbf{1}r_{f,s}\right)\right] = \mathbf{0}. \quad \text{(E.14)}$$

It follows from Eq. (E.14) that Eq. (E.13) holds with $\delta_3 = 2\left(1-\overline{\zeta}\right)\lambda_4$.

Third, assume that $C_s \geq C_{s,\alpha,\overline{\boldsymbol{w}}_E}$. It follows from Eq. (D.5) that the SCVaR weight is 0%. ■

Online Appendix F: Adding an SD constraint to our numerical example

Fig. F1. Relative change in the SD of the optimal portfolio due to an SD constraint

Consider an M-V objective function with risk aversion coefficient γ and an SD constraint with bound σ. Fig. F1 examines the relative change in the SD of the optimal portfolio due to this SD constraint. In each panel, the thick and thin dashed lines show the relative change as a function of $\gamma \in [0.1, 10]$ when the bound is, respectively, 2% and 3%. A risk-free asset is absent in panels A and C, whereas it is present in panels B and D. While the Volcker rule is present in panels A and B, it is absent in panels C and D. Relative changes are reported in percentage points.

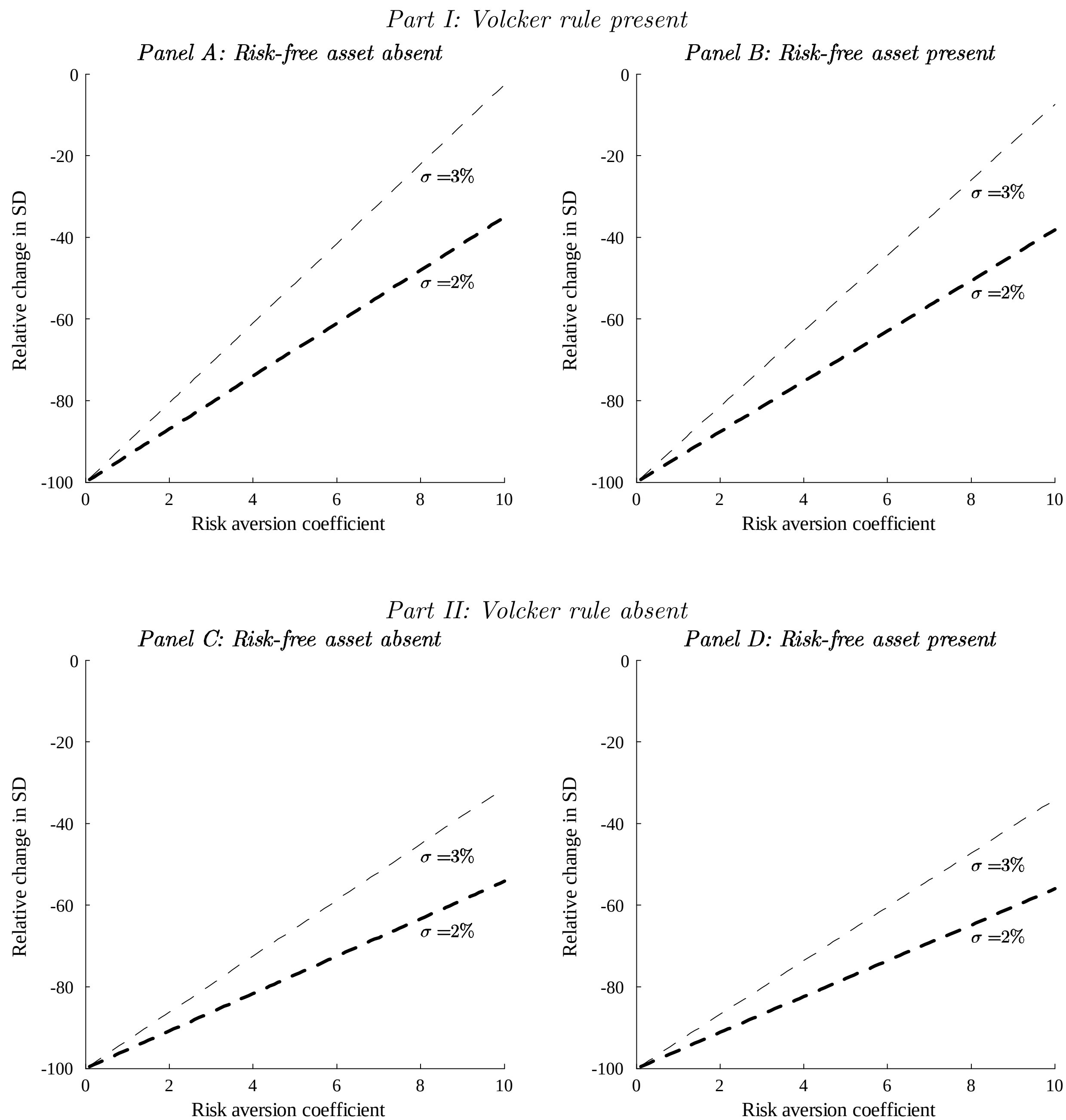


Fig. F2. **Relative change in the SCVaR of the optimal portfolio due to an SD constraint**

Consider an M-V objective function with risk aversion coefficient γ and an SD constraint with bound σ. Fig. F2 examines the relative change in the SCVaR of the optimal portfolio due to this SD constraint. In each panel, the thick and thin dashed lines show the relative change as a function of $\gamma \in [0.1, 10]$ when the bound is, respectively, 2% and 3%. A risk-free asset is absent in panels A and C, whereas it is present in panels B and D. While the Volcker rule is present in panels A and B, it is absent in panels C and D. Relative changes are reported in percentage points.

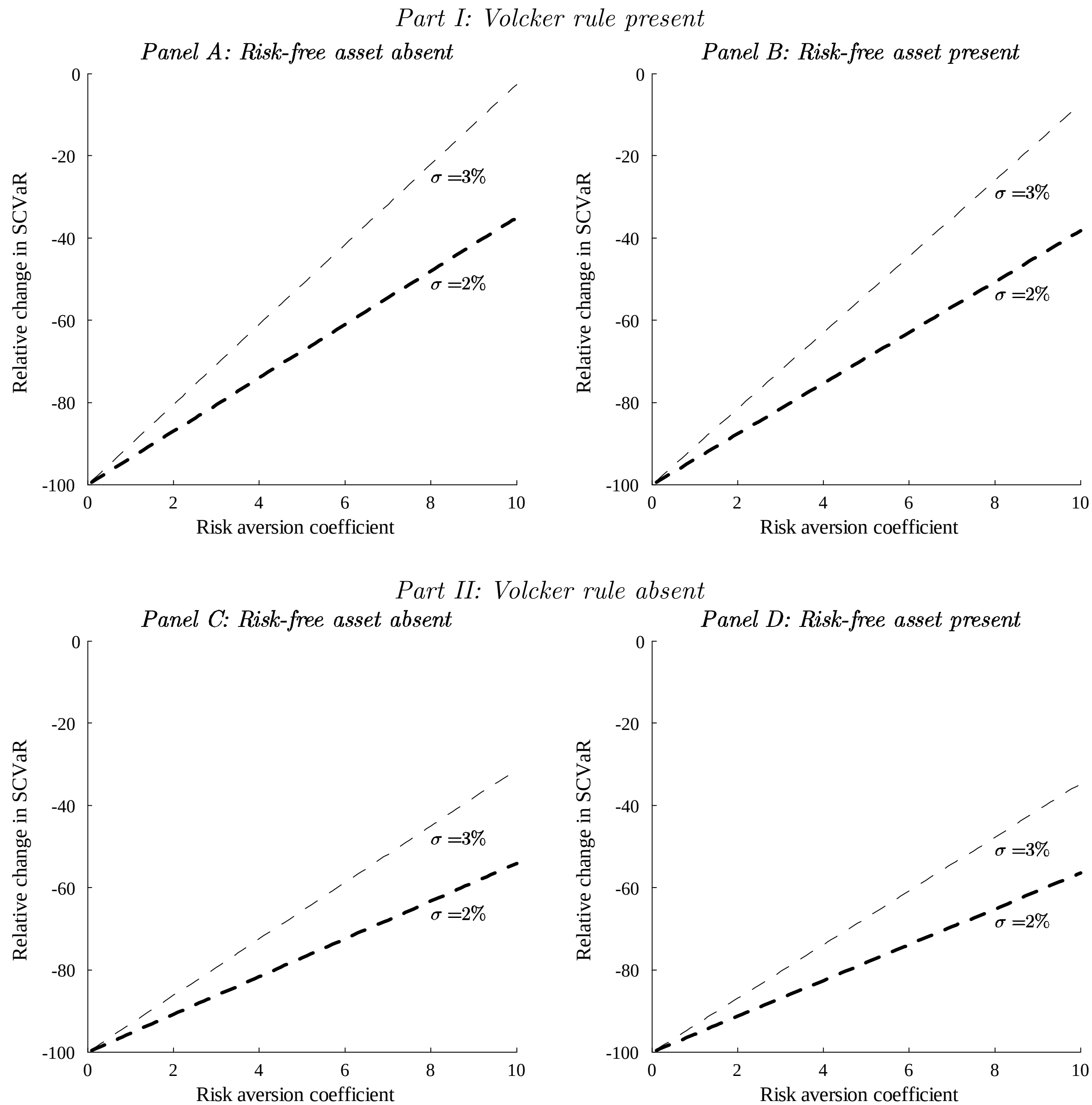


Fig. F3. Relative change in the CER of the optimal portfolio due to an SD constraint

Consider an M-V objective function with risk aversion coefficient γ and an SD constraint with bound σ. Fig. F3 examines the relative change in the CER of the optimal portfolio due to this SD constraint. In each panel, the thick and thin dashed lines show the relative change as a function of $\gamma \in [0.1, 10]$ when the bound is, respectively, 2% and 3%. A risk-free asset is absent in panels A and C, whereas it is present in panels B and D. While the Volcker rule is present in panels A and B, it is absent in panels C and D. Relative changes are reported in percentage points.